PROC. 15TH INT. CONF. PHYSICS OF SEMICONDUCTORS, KYOTO, 1980
J. PHYS. SOC. JAPAN **49** (1980) SUPPL. A p. 375–378

THEORY OF NEGATIVE MAGNETORESISTANCE IN DOPED SEMICONDUCTORS

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Theory of negative magnetoresistance in two-dimensional systems recently developed by Hikami et al. is extended to three dimension, and is applied to heavily doped semiconductors. The fundamental features of the experiments on GaAs and Ge are well explained. Anomalous positive magnetoresistance in p-type Ge is discussed, too.

I. Introduction

Negative magnetoresistance in doped semiconductors has been studied by many people for more than twenty years[1]~[8]. In spite of a considerable amount of experimental data, little progress has been made in the theory. Negative magnetoresistance has been observed in quasi-two-dimensional systems, e.g. in the inversion layer on Si surface[9], too, and recent developements in the theory of electron conduction in random systems have revealed that negative magnetoresistance is closely related to Anderson localization. In two-dimensional system of electrons in random potential, all the one electron eigenstates are localized irrespective of the strength of the potential[10]. This localization is the outcome of the time reversal symmetry of the system. Therefore, the eigenstates are delocalized if the time reversal symmetry is broken by magnetic field, and it gives rise to the increase in the conductivity, i.e. negative magnetoresistance. Recently Hikami et al.[11] worked out this problem by taking account of the contribution to the conductivity from the special type of multiple scattering of electrons by impurities shown in Fig.(1) in the form of the Feynman graph[12].

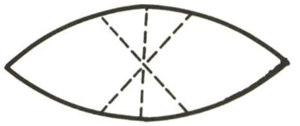

Fig.1 Feynman graph for the multiple scattering which contributes to negative magnetoresistance

In the present paper the theory will be extended to three-dimensional systems, and will be applied to doped semiconductors. In three-dimensional systems eigenstates are not localized if the random potential is weak enough, but the multiple scattering of Fig.(1) diminishes the conductivity by a considerable amount, and the suppression of this mechanism by magnetic field results in negative magnetoresistance. The interaction of electron spin with magnetic field is not important and will be neglected.

II. Theory

We start with a simple model of doped semiconductor, electron gas interacting with randomly distributed impurities. One electron

Hamiltonian is of the form

$$\mathcal{H} = (p + eA/c)^2/2m + \Sigma_R u\delta(r - R), \tag{2.1}$$

where u is the scattering amplitude of the impurities and R is their positions. Effects of inelastic scattering(electron-electron or electron-phonon) will be taken into account in the form of energy relaxation time[13]. The contribution to the conductivity from the multiple scattering in Fig.(1) has been calculated by the author[14], [15], and the conductivity is given by

$$\sigma = \sigma_0 + \Delta\sigma \ ; \quad \Delta\sigma = F(\delta)e^2/2\pi^2\hbar\ell, \tag{2.2}$$

where σ_0 is that without magnetic field, $\ell=(c\hbar/eH)^{1/2}$, H being the magnetic field, $\delta=\ell^2/4\tau_\varepsilon D$, with D the diffusion coefficient of electrons, and τ_ε is the energy relaxation time. The function $F(\delta)$ is given by

$$F(\delta) = \sum_{N=0}^{\infty} \{ 2(\sqrt{N + 1 + \delta} - \sqrt{N + \delta}) - \frac{1}{\sqrt{N + 1/2 + \delta}} \} . \tag{2.3}$$

The magnetoconductivity $\Delta\sigma$ is independent of the direction of the magnetic field relative to the current. The above expression is valid under the condition $\ell>>\lambda$, λ being the mean free path of electrons (this condition is fulfilled up to a few kOe in typical experiments). τ_ε is estimated to be proportional to T^{-3}(electron-phonon scattering) or to T^{-2} (electron-electron scattering), T being the temperature. At low temperature τ_ε becomes very large and we have $\delta<<1$ except in the regions of very low field. Then we put $F(\delta)=F(0)=0.605$, and obtain

$$\Delta\sigma = F(0)e^2/2\pi^2\hbar\ell = 0.918\sqrt{H} \text{ mho/cm (H in kOe) .} \tag{2.4}$$

Note that the above expression is independent of the electronic structure of the system. At high temperature we have $\delta>>1$. In this case $F(\delta)$ can be approximated as $F(\delta)=\delta^{-3/2}/48$, and hence

$$\Delta\sigma = \sigma_0(\tau_\varepsilon/\tau)^{3/2}(\omega_c\tau)^2/12\sqrt{3}, \tag{2.5}$$

with τ the momentum relaxation time of electrons due to the elastic scattering by the impurities, and ω_c the cyclotron frequency.

III. Case of Anisotropic effective mass.

Consider a case of spheroidal energy surface with longitudinal and transverse effective mass m_ℓ and m_t. By a simple transformation [15], the problem can be reduced to the case of isotropic effective mass $m_a=(m_\ell m_t{}^2)^{1/3}$ with the magnentic field H replaced by $H'=Hm_a/m_c$, m_c being the cyclotron mass. As a result, the magnetic field dependent part of the conductivity tensir is given by

$$\Delta\sigma_{\mu\nu} = \alpha_{\mu\nu}F(\delta_c)e^2/2\pi^2\hbar\ell_c \quad , \tag{3.1}$$

where $\delta_c=\ell_c/4\tau_\varepsilon D$, with $\ell_c=\ell(m_c/m_a)^{1/2}$, and $\alpha_{\mu\nu}=m_a(1/m)_{\mu\nu}$, $(1/m)_{\mu\nu}$being the effective mass tensor.

In the case of Ge and Si, the contributions from each valley are to be symply summed up if the intervalley scattering of electron is negligible[16]. Thus, at low temperature we obtain

$$\Delta\sigma_{\mu\nu} = S_{1/2}\times 0.918\sqrt{H} \text{ mho/cm (H in kOe),} \tag{3.2}$$

and at high temperature we obtain

$$\Delta\sigma_{\mu\nu} = \sigma_a(\tau_\varepsilon/\tau)^{3/2}(\omega_a\tau)^2 S_2/12\sqrt{3} \quad , \tag{3.3}$$

where $\sigma_a = ne^2\tau/m_a$, n being the electron density per one valley, $\omega_a = eH/cm_a$; the dependence on the direction of the magnetic field is given through S_x,

$$S_x = \sum_{\text{valley}} \alpha_{\mu\nu}(m_a/m_c)^x \quad . \tag{3.4}$$

IV. Comparison with Experiments

In the present theory, the effects of the impurities are taken into account perturbationally starting from free electron model, and hence it is to be compared with the data in metallic conduction regions. First we apply the theory of the case of isotropic effective mass to n-type GaAs. In Fig. (2) we show the data by Emeli'anenko and Nasledov[2]. Note that the dependence of $\Delta\sigma$ on the sample is much less than that of $\Delta\rho/\rho$. It is consistent with the theory, i.e., $\Delta\sigma$ should not depend on the sample in the limit of low temperature. Equation (2.4) gives about twice as large value, and it can be attributed to that the temperature is not low enough. For the sample No.2, the best fit in the low field regions was obtained by putting $\tau_\varepsilon = 0.8\times10^{-11}$sec. This value is reasonable if we assume electron-phonon interaction. The deviation in the high field regions is due to the violation of the condition $\ell \gg \lambda$ and to the normal positive magnetoresistance proportional to H^2.

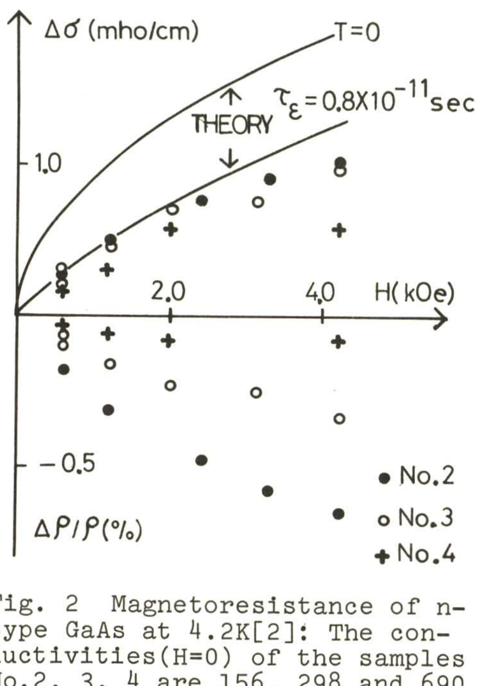

Fig. 2 Magnetoresistance of n-type GaAs at 4.2K[2]: The conductivities(H=0) of the samples No.2, 3, 4 are 156, 298 and 690 mho/cm, respectively

Next we consider n-type Ge. Dependence of $\Delta\rho/\rho$ on H has been carefully studied by Roth et al. [4]. They analized their data on the assumption

$$\Delta\rho/\rho = aH^c + bH^2, \tag{4.1}$$

and found that c tends to 1/2 at low temperature in the samples of high donor concentration (see Table (1)). This is consistent with eq. (3.2).

$N_D(10^{19}\text{cm}^{-3})$	c 4.2K	c 1.8K
0.13	0.98	0.72
0.30	0.75	0.57
0.88	0.62	0.51
1.23	0.63	0.51

Table 1 The exponent c of the field dependence of the magnetoresistance for the samples of various donor concentration N_D(cf. eq. (4.1)).

Anisotropy of magnetoresistance in n-type Ge has been studied extensively by Furukawa[3] and also by Sasaki[5]. In Fig. 3 we show the dependence of $\Delta\rho/\rho$ on the direction of the magnetic field measured by Sasaki in the regions where $\Delta\rho \propto H^2$. From eq.(3.4) we get

$$S_2 = 27.9(1 - .29\cos^2\theta) \tag{4.2}$$

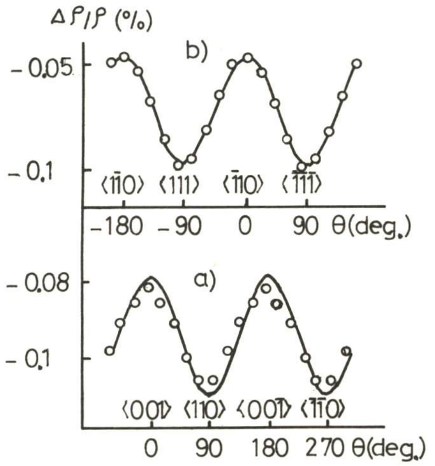

Fig. 3 Anisotropy of magnetoresistance in n-type Ge[5]: a) H ⊥ ⟨1̄10⟩ , I // ⟨1̄10⟩ , and b) H ⊥ 112̄ , I // ⟨1̄10⟩ (— Theory, o experiment)

for case (a), and

$$S_2 = 25.3(1 - .53\cos^2\theta) \quad (4.3)$$

for case (b). If we put $\Delta\rho/\rho = AS_2$ with A an adjustable constant, the anisotropy is well reproduced as is seen in Fig. (3).

V. Effects of Spin-Orbit Coupling

Hikami et al.[11] showed that the sign of $\Delta\sigma$ is reversed if spin-orbit coupling is strong so that spin flip relaxation time τ_s is shorter than the energy relaxation time τ_ε. This is the case in three dimension, too. Anomalous positive magnetoresistance has been observed in p-type Ge [4], [8]. Sugiyama and Kobayashi observed that $\Delta\rho$ becomes negative under uniaxial stress. In the valence band of Ge spin-orbit coupling must have a considerable value and there is a possibility that τ_s is short enough to give rise to the above mentioned effect. Under uniaxial stress the four fold degeneracy of the valence band is lifted, and τ_s becomes longer because the contribution of interband transtion is much larger than that of intraband transition. Because of the complexity of the band structure, quantitative analysis has not yet been done.

The author is grateful to Prof. S.Kawaji for valuable discussions.

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