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CONDUCTIVITY AND HALL EFFECTS IN TWO-DIMENSIONAL DISORDERED SYSTEMS

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The precursor effect of the Anderson localization in two-dimensional metals, i.e. the logarithmic correction term in the transport coefficients, has been examined microscopically for conductivity, Hall coefficient and Seebeck coefficient. Present investigations include the effects of the intervalley impurity scattering and the mutual interactions between electrons. The results can explain recent experimental findings in Si-MOS.

I. Introduction

Recent scaling theories by Abrahams et al. [1] and Anderson et al. [2] indicate that two-dimensional metals are not truly metallic in the sense that the conductivity, σ, will not tend to a finite value in the low temperature limit, but that σ will always be reduced as the temperature is lowered, logarithmically at first and exponentially at low temperatures. The logarithmic corrections to the metallic conductivity is given by

$$\sigma = \sigma_0 \left[1 - \frac{\alpha'}{2\pi\varepsilon_F\tau_0} \ell n \frac{\tau_\varepsilon}{\tau_0}\right] , \tag{1}$$

where $\sigma_0=\varepsilon_F\tau_0 e^2/\pi$ and α' is some numerical constant of order unity. Here ε_F, τ_0 and τ_ε are the fermi energy, the relaxation time due to impurity scattering and that due to inelastic scattering, respectively. Later, microscopic calculations by Gorkov et al. [3] and by Khmelnitzkii [4] based on diagram technique confirmed the result of eq. (1). In their works processes contributing to the conductivity are systematically analysed as a perturbational series with respect to $(\varepsilon_F\tau_0)^{-1}$, whose leading term is the result of the Boltzmann transport equation, σ_0, and the first correction is seen to include the singular contribution at low temperatures. The numerical constant, α', is shown to be universal, $\alpha'=1$, in these investigations.

Experimentally the existence of such logarithmic correction to the conductivity has been demonstrated in metallic films [5] and in MOS [6]. In the case of MOS, Bishop, Tsui and Dynes [6] observed the logarithmic increase of resistivity below $T\sim1$K even for electron density of $n=5.6\times10^{12}\mathrm{cm}^{-2}$, which used to be classified as in metallic regime. By assuming that τ_ε in eq. (1) is inversely proportional to some power, p, of temperature, $\tau_\varepsilon\propto T^{-p}$, they concluded $\alpha' p/2=0.52\pm0.05$. On the other hand Kawaguchi and Kawaji [7] investigated this problem by use of magnetoresistance. They analyzed the experimental data for samples with similar electron densities to those of Bishop et al. based on the theoretical result by Hikami, Larkin and Nagaoka [8], who examined the effect of magnetic fields on the second term of eq. (1) and found anisotropic negative magnetoresistance. In this investigation, the magnetic field dependence of the conductivity is fitted between theory and experiment (fitting was done excellently) and α' and τ_ε are determined separately. Their results indicate that $\tau_\varepsilon \propto T^{-p}$ with $p \lesssim 2$ and $\alpha'/n_v \equiv \alpha = 0.25 \sim 0.35$, where n_v is the valley degeneracy and $n_v=2$ for (1,0,0) surface of the experiment. Thus these experiments on MOS are consistent with each other, and confirm the existence of the precursor effect of the Anderson localization in two-dimension, i.e. logarithmic correction to the conductivity in samples so far classified as metallic. The prefactors of the logarithmic corrections in these experiments, however, are definitely smaller than the theoretical expectation of $\alpha=1$.

In this paper we will resolve this discrepancy by taking into account of intervalley impurity scattering and the mutual interactions.

II. Effects of Intervalley Impurity Scattering

In the presence of n_v equivalent valleys we will write the logarithmic correction to conductivity as

$$\alpha' = -\frac{e^2}{2\pi^2} n_v \alpha \ln \frac{\tau_\varepsilon}{\tau_0} \quad . \tag{2}$$

In Si(1,0,0) surface there exist two equivalent valleys. If these two valleys are independent, we have $n_v=2$ and $\alpha=1$. In the presence of intervalley impurity scattering, whose scattering rate is defined as $1/2\tau'$, σ' is given as follows [9],

$$\sigma' = -\frac{e^2}{\pi^2}\left[\frac{1}{2}\ln\frac{\tau_\varepsilon}{\tau_v} - \ln\left(\frac{\tau_0}{\tau'} + \frac{\tau_0}{\tau_\varepsilon}\right)\right] \quad , \tag{3}$$

where $\tau_v=(1-2\tau_0/\tau'-\tau_0/\tau_\varepsilon)/(2/\tau'+1/\tau_\varepsilon)$. If $\tau' \ll \tau_\varepsilon$ the second term of eq. (3) does not have $\ln\tau_\varepsilon$ dependence, and the first term leads to a conclusion $n_v\alpha=1$, i.e. $\alpha=1/2$. On the other hand we recover $n_v\alpha=2$ once $\tau_\varepsilon \ll \tau'$. Thus we can expect a change of α in the temperature region of $\tau_\varepsilon \sim \tau'$.

We also examined the case of Si(1,1,1) surface. In this case, $n_v=6$ and we will have $n_v\alpha=6$ if each valley is treated as independent. However we found that $n_v\alpha=1$ also in this case if the intervalley impurity scattering is more frequent than the inelastic scattering, which is possible in realistic systems [10]. Actually the experiment by Kawaguchi et al. [11] on cesiated Si(1,1,1) surface demonstrates the logarithmic temperature dependence of resistivity whose prefactor is consistent with $n_v\alpha=1$.

III. Effect of Mutual Interactions [12]

We considered the case of single kind of carriers and the effect of four characteristic types of interactions, g_i, shown in Fig.1 for

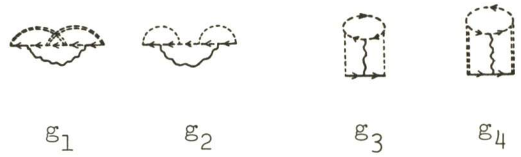

Fig.1 Contribution to the self-energy function in the linear order of the mutual interactions: The broken and the double broken lines represent particle-hole and particle-particle diffusion processes

self-energy functions where broken and double-broken lines are sum of the ladder (i.e. diffusion) processes of particle-hole and particle-particle. These processes lead to singular temperature dependence in the relaxation time (in contrast to the case of non-interacting systems) and in the conductivity.

$$\frac{1}{2\tau} = \frac{1}{2\tau_0} \left[1 + \frac{g}{2\pi\varepsilon_F\tau_0} \ln \frac{1}{4\pi\tau_0 T}\right] , \qquad (4)$$

$$\sigma = \sigma_0 \left[1 - \frac{1}{2\pi\varepsilon_F\tau_0} \ln\frac{\tau_\varepsilon}{\tau_0} - \frac{g}{2\pi\varepsilon_F\tau_0} \ln \frac{1}{4\pi\tau_0 T}\right] , \qquad (5)$$

where $g=g_1+g_2-2(g_3+g_4)$. Since only the particle-particle diffusion process is sensitive to magnetic fields, logarithmic contributions except those from processes with g_1 and g_3 are suppressed once $\omega_c\tau_0>(\varepsilon_F\tau_0)^{-1}$ ($\omega_c=eH/mc$), i.e.

$$\sigma = \sigma_0 \left[1 - \frac{g_1-2g_3}{2\pi\varepsilon_F\tau_0} \ln \frac{1}{4\pi\tau_0 T}\right] , \ \omega_c\tau_0>(\varepsilon_F\tau_0)^{-1} . \qquad (6)$$

Thus we see that mutual interactions affect the prefactor of the logarithmic correction of conductivity and that the presence of g_1 and g_3 leads to the difference in this prefactors deduced from the temperature dependence on one hand and the magnetic field dependence at a fixed temperature on the other hand.

Effect of mutual interactions on the density of states and the conductivity have also been examined by Altschuler et al. [13], who considered the processes corresponding to g_1 and g_3 in the present notation. They concluded that g_1 is more important than g_3. Instead of energy and momentum independent coupling constant g_1 they considered that this is given by Coulomb interactions screened by polarization process of diffusion type. By this treatment they concluded that $g_1=1$, i.e. independent of e^2, and that this can be the origin of the logarithmic temperature dependence of conductivity observed in the experiment [6]. As is discussed, this process is insensitive to magnetic fields and then magnetoresistance will not be present in this treatment in contrast to the experimental findings [7].

IV. Hall Coefficient

In order to understand further the implication of the logarithmic correction, we evaluated the Hall conductivity [14], σ_{xy}, in the same context. By use of the diagram technique to σ_{xy} [15], we found that

$$\sigma_{xy} = \sigma_0\omega_c\tau_0 \left[1 - 2\times\frac{1}{2\pi\varepsilon_F\tau_0} \ln \frac{\tau_\varepsilon}{\tau_0}\right] , \qquad (7)$$

and that the effective carrier number, n_{eff}, deduced from the Hall coefficient, $R=\sigma_{xy}/H\sigma^2=(n_{eff}ec)^{-1}$, is equal to the actual carrier number, n. This result is confirmed by Altshuler et al. [16]. Thus the logarithmic correction is considered to be in the effective relaxation time. This fact remains valid even if intervalley impurity scatterings are taken into considerations [9].

Experimentally the Hall coefficient is found to be [17] constant in the temperature region where the conductivity has logarithmic dependence, in accordance with present theoretical result.

V. Seebeck Coefficient

The Seebeck coefficient (isothermal thermoelectric power), S, is given by $S=\beta/\sigma T$ where β is the correlation function between heat current and the electric current. Microscopic calculations of this correlation function shows that β does not have leading logarithmic correction and then we have

$$S/S_0 = \sigma_0/\sigma ,$$

where $S_0=\pi^2T/3e\varepsilon_F$ is the result of the Boltzmann transport equation.

VI. Conclusion

Recent experiments on the temperature dependences of the conductivity, the magnetoresistance and the Hall effect in Si-MOS have observed the precursor effect of Anderson localization in two-dimension. The correspondence between theory and experiment is seen to be satisfactory if the intervalley impurity scattering is taken into account. The possibility of roles (but relatively minor) played by mutual interactions is discussed.

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References

1) E. Abrahams, P.W. Anderson, D.C. Licciardello and T.V. Ramakrishnan: Phys. Rev. Lett. 42 (1979) 673.
2) P.W. Anderson, E. Abrahams and T.V. Ramakrishnan: Phys. Rev. Lett. 43 (1979) 718.
3) L.P. Gorkov, A.I. Larkin and D.E. Khmelnitzkii: J.E.T.P. Lett. 30 (1979) 248.
4) D.E. Khmelnitzkii: NORDITA preprint.
5) G.J. Dolan and D.D. Osheroff: Phys. Rev. Lett. 43 (1979) 721.
6) D.J. Bishop, D.C. Tsui and R.C. Dynes: Phys. Rev. Lett. 44 (1980) 1153.
7) Y. Kawaguchi and S. Kawaji: J. Phys. Soc. Jpn 48 (1980) 699.
8) S. Hikami, A.I. Larkin and Y. Nagaoka: Prog. Theor. Phys. 63 (1980) 707.
9) H. Fukuyama: J. Phys. Soc. Jpn 49 (1980) #2.
10) T. Ando: Proc. Yamada Conf. II on Electronic Properties of 2D Systems. (Lake Yamanaka, 1979), Surface Sci. 98 (1980).
11) Y. Kawaguchi, H. Kitahara and S. Kawaji: Surf. Sci. 73 (1978) 520.
12) H. Fukuyama: J. Phys. Soc. Jpn 48 (1980) 2169.
13) B.L. Altschuler, A.G. Aronov and P.A. Lee: Phys. Rev. Lett. 44 (1980) 1288.
14) H. Fukuyama: J. Phys. Soc. Jpn 49 (1980) #2.
15) H. Fukuyama, H. Ebisawa and Y. Wada: Prog. Theor. Phys. 42 (1969) 494.
16) B.L. Altchuler, D.E. Khmelnitzkii, A.I. Larkin and P.A. Lee: preprint.
17) Y. Kawaguchi and S. Kawaji: private communications.