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MONTE CARLO STUDY ON DYNAMICAL ASPECTS OF THE 2D GAUSSIAN ISING SPIN GLASS

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Dynamical aspects of the 2D Gaussian Ising spin glass are studied by the Monte Carlo simulation. It is found that the spin-freezing process in the system at temperatures $T \gtrsim 0.8\bar{J}$ is well described by Fulcher's law with the characteristic temperature $T_0 \cong 0.4\bar{J}$, $\bar{J}$ being the width of the distribution of J_{ij}. Physical relevance of this result is discussed in connection with the frequency-dependence of ac susceptibilities and the remanence in magnetizations.

1. Introduction

It is now well recognized that proper understandings of dynamical properties of spin glass (SG) are indispensable to solve the SG problem. In our previous work [1,2], hereafter referred to as I and II, we investigated dynamical (time-dependent) aspects of individual spins and spin clusters in the two-dimensional (2D) $\pm J$ Ising SG simulated by the standard Monte Carlo (MC) method. We found that the spin-freezing process in the system at least at temperatures $T \gtrsim 0.8J$ obeys Fulcher's law [3-5], which we ascribed to cooperative behavior of spin clusters (including the way how the clusters themselves develop as the temperature decreases).

In this communication we report results of similar MC study on 2D Gaussian Ising SG (ISG), in which the nearest neighbor interactions J_{ij} are distributed following the probability $P(J_{ij})=\exp(-J_{ij}^2/2\bar{J}^2)$, $\bar{J}$ being the width. As in the $\pm J$ ISG, it is found that the spin-freezing process in this system at least at temperatures $T \gtrsim 0.8\bar{J}$ also obeys Fulcher's law with the characteristic temperature $T_0 \cong 0.4\bar{J}$. This result, combined with our previous one, strongly suggests that Fulcher's law is a common property of SG's with short-ranged interactions (Sec.2). Physical relevance of this result is discussed in connection with the frequency-dependence of ac susceptibilities and the remanence in magnetizations (Sec.3).

2. Fulcher's law

The method as well as the system (25×25 with the periodic boundary condition) of the present MC study are almost identical to those in our previous work I except for the distribution of J_{ij}. To investigate the spin-freezing process in the system it is desirable to have its microscopic informations, i.e., the relaxation time τ_i of each spin σ_i. For this purpose we first evaluate the auto-correlation function $\langle\sigma_i(0)\sigma_i(t)\rangle$ for $t \leqq 3000$ MC steps per spin (MCS) by making use of a MC run of 60000 MCS at each temperature. We then fit the results to the following double exponential function

$$\langle\sigma_i(0)\sigma_i(t)\rangle \cong p_{Ii}\exp(-t/\tau_{Ii}) + p_i\exp(-t/\tau_i) \tag{1}$$

with $p_{Ii}+p_i=1$. Here we have assumed that the longer relaxation time τ_i of our primary interest represents dynamical aspect of the spin cluster, to which the i-th spin belongs, while the shorter τ_{Ii} represents the individual thermal excitation (and de-excitation) of the spin which occurs many times even in the time period of τ_i. This fitting works rather well at temperatures $0.8\bar{J} \lesssim T \lesssim 1.5\bar{J}$ also for the present Gaussian ISG, and τ_i can be determined for more than 85% of spins. Making use of τ_i thus obtained, we determine for each spin at each temperature the effective barrier energy $E_i(T;T_0)$ by the equation

$$\tau_i = \tau_0 \exp \{ E_i(T;T_0)/(T-T_0) \} \tag{2}$$

where τ_0 is put 1-MCS which is a unique intrinsic time scale in the present MC simulation. We then examine the distribution of $E_i(T;T_0)$.

The gross feature of E_i-distribution becomes T-independent if we put $T_0 \cong 0.4\bar{J}$. Alternatively we plot in Fig. 1 the average of $\log\tau_i$ over whole spins, whose σ_i has been specified. From these results we claim that the spin-freezing process in this system at $T \gtrsim 0.8\bar{J}$ obeys Fulcher's law with the

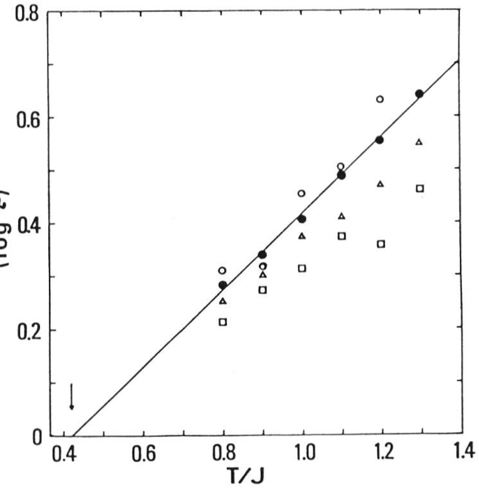

Fig. 1: The inverse plot of the average of $\log\tau_i$ over whole spins (●) and that of $\log\tau$ of some typical spin clusters (○, △, □). The solid line indicates Fulcher's law with $T_0 = 0.42\bar{J}$ and $\bar{E}_i = 3.2\bar{J}$.

characteristic temperature $T_0 \cong 0.4\bar{J}$. (We have done these analyses on two samples having different sets of J_{ij}, and obtained almost identical T_0's.) In Fig. 1 we also plot T-dependences of τ of some typical spin clusters, where τ is the average of τ_i over spins in the cluster. It is seen that the relaxation process of each cluster is also described by Fulcher's law with the same T_0. This feature, also found in $\pm J$ ISG (Fig. 2 in II), is interpreted as the cooperative behavior of spin clusters. Actually detailed inspection of a sequence of spin pattern reveals that cooperative aspects of spin clusters in the present system are similar to those in $\pm J$ ISG explained in I and II. There exist, of course, certain intrinsic differences in the clustering features in the two ISG's. In $\pm J$ ISG a minimum unit of clusters are 4 spins surrounding an unfrustrated square, while clusters of a pair spins appear where $|J_{ij}|$ are extremely large in Gaussian ISG. In the ground states aliving spins [6] do exist in $\pm J$ ISG, but vanish in Gaussian ISG (because J_{ij} are distributed continuously). These differences, however, do not affect on the gross feature of Fulcher's law in the temperature region of the present interest. This strongly suggests that Fulcher's law commonly describes the spin-freezing process in SG's with short-ranged interactions.

3. Discussions

To examine physical relevance of our results obtained in the previous section, we first calculate ac susceptibility $\chi(\omega)$ by means of the equation

$$\chi(\omega) = \frac{1}{NT} \sum_i \left(\frac{p_{Ii}}{1 + i\omega\tau_{Ii}} + \frac{p_i}{1 + i\omega\tau_i} \right) \qquad (3)$$

from our data p_{Ii}, p_i, τ_{Ii} and τ_i obtained through Eq.(1). Equation (3) is used in the cluster-model approach to the SG problem [7], which is here empirically extended to individual spins having local susceptibility $\chi_i=1/T$. Note that its first term represents possible non-vanishing contribution to $\chi(\omega)$ even if the cluster, to which the i-th spin belongs, is already frozen ($\omega\tau_i>>1$). (In a similar analysis in I we neglected this first term and put $p_i=1$. This results in a very sharp drop of $\chi(\omega)$ just below the cusp temperature, although the value of the latter itself does not change significantly by the present modification.) The calculated results are shown in Fig. 2, where we plot for comparison the susceptibility determined from the

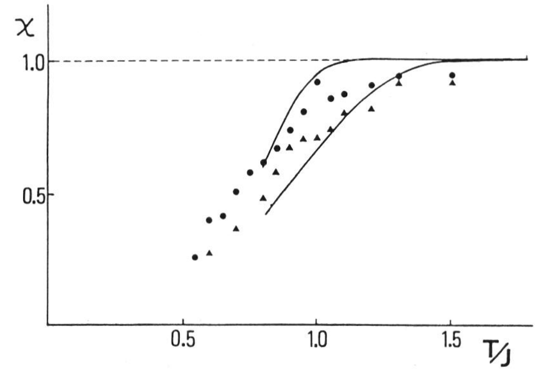

Fig. 2: Susceptibilities calculated by the standard MC method with the observation periods of 10^3 (▲) and 10^4 MCS (●). The lower and upper lines indicate the corresponding $\chi(\omega)$ evaluated by Eq. (3), respectively.

variance of the magnetization in the corresponding MC observation period (MCS=$2\pi/\omega$). The coincidence of the two χ's is satisfactory, indicating that the spin auto-correlation function obtained here accounts properly the spin fluctuation in the system. Also we can reread T and τ (of a whole system) in Fig. 1, respectively, as the cusp (freezing) temperature T_g and inverse of the frequency of the ac measurement, by which T_g is determined [4].

Up to this point we have examined behavior of the SG system under zero magnetic field. We now investigate the remanence in magnetizations, in particular, thermo-remanent-magnetization (TRM), i.e., first the system is cooled under a finite magnetic field H, and we look at the total magnetization (per spin) M just after the field is switched off at a certain low temperature. This process can be literally followed by the MC simulation as has been already done by Kinzel [8]. In the present analysis the system is cooled extremely gradually; It is cooled with the temperature intervals of $0.5\sim1.0\bar{J}$ and at each temperatures 15000 MCS are used for equilibration, making observation of various quantities at the same time as in I. It is noted here that the field-cooled M thus obtained increases as temperature decreases and becomes constant at lower temperatures in case $H\gtrsim0.2\bar{J}$. A knee-like transition in M [9] has been observed clearly, but only under larger fields $H\gtrsim0.5\bar{J}$. These details will be reported elsewhere. In Fig. 3 we show TRM obtained by our simulation in case $H=\bar{J}$. The result at $T=0.2\bar{J}$ agrees with Kinzel's one. We have tried to calculate TRM by means of the spin auto-correlation functions of all spins evaluated before (under H=0), using, as the initial spin configuration, the one appeared in the MC run just when the field is switched off. Since the distribution of τ_i of the present system becomes

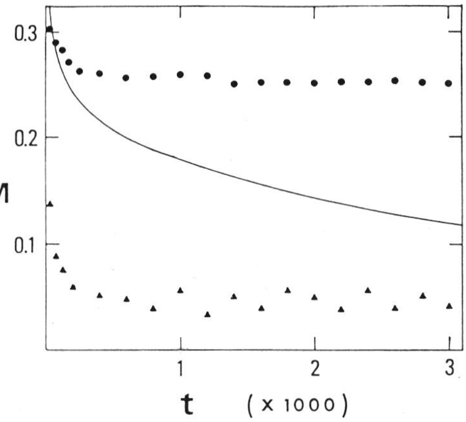

Fig. 3: TRM simulated by the MC method with $H=\bar{J}$ at $T=0.2\bar{J}$ (●) and $0.8\bar{J}$(▲). Just when H is switched off, $M(t=0)\cong 0.535$ in both cases. The solid line represents our calculated TRM at $T=0.8\bar{J}$ as explained in the text.

very broad at lower temperatures, we can expect a certain remanent behavior in M thus calculated. It is in fact the case and this M, also shown in Fig. 3, exhibits a logarithmic dependence on t in the time interval of $1000 \lesssim t \lesssim 3000$ MCS. However, it does not reproduce the TRM obtained by the direct MC simulation. As seen in Fig. 3, the latter relaxes much more rapidly than our calculated TRM. At the moment our data are not sufficient to make clear whether this discrepancy is qualitative or only quantitative. (The relaxed M ($\cong 0.5$) in the MC simulation can be certainly specified as the TRM, but its t-dependence can not be well analysed within the accuracy of our present simulation.) Further analyses on remanent magnetizations are now undertaken.

The numerical calculation was made by Hitac M-200H at Hokkaido University Computing Center.

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