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DYNAMICS OF KINKS, INTERFACES, VORTICES AND KINETICS OF PHASE TRANSITIONS

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A method is presented to derive stochastic equations of motion of kinks, interfaces and vortex lines from underlying nonlinear stochastic model field equations. The results are used to study kinetics of fluctuation in systems quenched to thermodynamically unstable states.

1. Introduction

In recent years importance of topological singularities such as kinks, interfaces, vortex lines, etc. has come to be widely recognized[1]. We here present a method to derive equations of motion obeyed by such singularities, and we then illustrate usefulness of the approach in kinetics of first order phase transitions.

2. Kinks

We explain the method for the following one-dimensional stochastic TDGL equation

$$\dot{y}(x,t) = -L(\partial_x)\frac{\delta H(t)}{\delta y(x,t)} + f(x,t) \tag{2.1}$$

where the random force arising from thermal noise f(x,t) obeys the fluctuation -dissipation relation,

$$\langle f(x,t)f(x',t')\rangle = 2k_BTL(\partial_x)\delta(x-x')\delta(t-t') \tag{2.2}$$

Here a dot denotes time derivative, $\partial_x \equiv \partial/\partial x$ and $L(\partial_x)$ is a self-adjoint positive definite differential operator. H is a coarse-grained free energy functional having the following general form:*)

$$H = \int dx\left\{\frac{1}{2}(\partial_x y)^2 + gV(y)\right\} \tag{2.3}$$

The explicit form of V(y) need not be given but we assume that H has a kink-type stationary state,

$$\frac{\delta H}{\delta y} = \left[-\partial_x^2 + gV'(y)\right]y = 0 \tag{2.4}$$

whose solution $y=M_i(x-x_i)$ gives an isolated kink profile centered at $x=x_i$. We are interested in the case of small kink width $g\gg1$. In the presence of many kinks, superpositions of kink solutions are no longer stationary unless constrained by conservation laws, but the kinks start to move due to kink-kink interactions.

Derivation of equations of motion of kinks is conveniently carried out by means of a variational formulation of (2.1) which takes the form

$$-\frac{\delta H}{\delta y} = \frac{\delta F}{\delta \dot{y}} - \frac{\delta \Delta F}{\delta f}. \tag{2.5}$$

*) In the following we often omit arguments of functions whenever no confusion arises.

Here F is the Rayleigh dissipation functional given by

$$F\{\dot{y}\} = \frac{1}{2}\int \dot{y}L^{-1}\dot{y}\,dx \tag{2.6}$$

and ΔF is the fluctuation functional defined by $\Delta F=F\{f\}$. Substituting into F, H, ΔF the following ansatz for y near $x=x_i$ which is a superposition of independent kinks which are numbered from left to right, we obtain

$$y = M_i(x-x_i) + \sum_{j>i}[M_j(x-x_j)-M_j(-\infty)] + \sum_{j<i}[M_j(x-x_j)-M_j(\infty)]\,. \tag{2.7}$$

Here and after we take g to be sufficiently large so that kink widths are negligibly small compared with distances between kinks. We thus find $\dot{y}=-\Sigma\dot{x}_i M_i'$ with $M_i' \equiv dM_i/dx$ and

$$F\{\dot{y}\} \rightarrow F_K\{\dot{x}\} \equiv \frac{1}{2}\sum_{ij}(M_i', L^{-1}M_j')\dot{x}_i\dot{x}_j \tag{2.8}$$

where K stands for kink and $(X,Y)\equiv\int X(x)Y(x)dx$. Similary, with $f=-\Sigma\theta_i M_i'$, θ_i being a random force acting on the i-th kink, we find

$$\Delta F\{f\} \rightarrow \Delta F_K\{\theta\} = \frac{1}{2}\sum_{ij}(M_i', L^{-1}M_j')\theta_i\theta_j \,. \tag{2.9}$$

H{y} also reduces to $H_K\{x_i\}$ which is the potential energy of kink interactions. In this way (2.5) can be converted into the kink equation of motion as

$$-\frac{\partial H_K}{\partial x_i} = \frac{\partial F_K}{\partial \dot{x}_i} - \frac{\partial \Delta F_K}{\partial \theta_i} \,. \tag{2.10}$$

The left hand side, the force acting on the i-th kink, is evaluated in Appendix. When we have one type of kinks and antikinks, (2.10) gives [2]

$$\sum_j (M_i', L^{-1}M_j')(\dot{x}_j - \theta_j) = -g\left[\delta M(x_{i+1}-x_i) + \delta M(x_{i-1}-x_i)\right]\Delta M\, V'' \tag{2.11}$$

where $\Delta M\equiv M(\infty)-M(-\infty)$, $V''\equiv V''(M(\pm\infty))$, $\delta M(x)\equiv M(x)-M(\infty\times\mathrm{sgn}(x))$ and $M(x)$ is the kink profile centered at x=0. $\delta M(x)$ is a small exponential tail of the kink profile. The fluctuation-dissipation relation for θ_i can be found by noting that the path probability of random forces is proportional to $\exp[-\int\Delta F_K(t)dt/2k_BT]$ as

$$\langle\theta_i(t)\theta_j(t')\rangle = 2k_BT[(M', L^{-1}M')^{-1}]_{ij}\,\delta(t-t') \tag{2.12}$$

where $(M', L^{-1}M')$ is a matrix whose ij-element is $(M_i', L^{-1}M_j')$. Appendix contains an extension of the method to a model with an inertial term.

A particularly simple case obtains when L is a constant. The kink equation of motion then assumes the following form in suitable units [2];

$$\dot{x}_i = R(x_{i+1}-x_i) - R(x_i - x_{i-1}) + f_i \tag{2.13}$$

$$\langle f_i(t)f_j(t')\rangle = 2k_BT\delta_{ij}\delta(t-t') \tag{2.14}$$

where $R(x)=e^{-x/\xi}$ is the attractive force between adjoining kinks, ξ being the kink width. This equation can describe the growth of domains in quenched effectively one-dimensional systems without conservation law if (2.13) is supplemented with instantaneous annihilation processes of kinks and antikinks upon contacts. Here a relevant quantity is the average domain size $\ell(t)$ which is proportional to the Bragg scattering intensity [3] and is inversely proportional to the number density of kinks. If thermal noise predominates over attractive forces in (2.13) we have $\ell(t)\propto t^{1/2}$ by diffusion. If attractive forces are dominant, we expect $\ell(t)\propto\ln t$. This latter result is obtained as follows. If in the absence of interactions all the kinks move with the constant speed $\bar{v}$, the kink annihilation rate will be about $\bar{v}\ell(t)^{-2}$ which leads to $\ell(t)\propto t$. In the presence of interactions, however, (2.13) gives $\bar{v}\exp[-\ell(t)/\xi]$ for the average relative velocity of adjacent kinks, reducing the annihilation rate by the factor $\exp[-\ell(t)/\xi]$. Solving the resultant self-consistent equation for $\ell(t)$ leads to $\ell(t)\propto \ln t$. This behavior seems to be supported by experiments [3]. See also the note added at the end.

3. Interfaces

We now extend the approach of Sec.1 to the following TDGL equation for scalar order parameter $S(\mathbf{r},t)$ of higher spatial dimensionality:

$$\frac{\partial}{\partial t}S(\mathbf{r},t) = -L(\nabla)\frac{\delta H\{S(t)\}}{\delta S(\mathbf{r},t)} + f(\mathbf{r},t) \tag{3.1}$$

where

$$H\{S\} = \int [\tfrac{1}{2}(\nabla S)^2 + g V(S)]\,d\mathbf{r} \tag{3.2}$$

$$\langle f(\mathbf{r},t) f(\mathbf{r}',t')\rangle = 2k_B T L(\nabla)\delta(\mathbf{r}-\mathbf{r}')\delta(t-t') \tag{3.3}$$

$L(\nabla)$ being a positive definite self-adjoint differential operator. The variational formulation of (3.1) is a straightforward extension of (2.5) and (2.6):

$$-\frac{\delta H}{\delta S} = \frac{\delta F}{\delta \dot{S}} - \frac{\delta \Delta F}{\delta f} \quad (3.4); \qquad F\{\dot{S}\} \equiv \tfrac{1}{2}\int \dot{S}\,L(\nabla)^{-1}\dot{S}\,d\mathbf{r}\,, \quad \Delta F \equiv F\{f\} \tag{3.5}$$

Instead of kinks we now have interfaces. Only new feature here is that a single interface is stationary only in the absence of mean curvature (minimal surface) where we have $\delta H/\delta S=0$. This gives the order parameter profile $M(z-z(a))$ near the interface $z=z(a)$ where z is a curvilinear coordinate in normal direction to the interface where a denotes coordinates on the interface. Thus we have

$$\nabla S = \mathbf{n}(a)M'(z-z(a))\,, \quad \dot{S} = -v(a)M'(z-z(a))\,, \quad f = -\theta(a)M'(z-z(a)) \tag{3.6}$$

where $\mathbf{n}(a)$ is the unit normal vector of the interface and $v(a)$ is the normal component of the interfacial velocity. (3.5) now reduces to

$$F\{\dot{S}\} \to F_{DH}\{v\} = \tfrac{1}{2}\iint da\,da'\langle a|\Gamma^{-1}|a'\rangle v(a)v(a') \tag{3.7}$$

and similarly for ΔF where Γ^{-1} is a matrix whose aa'-element is

$$\langle a|\Gamma^{-1}|a'\rangle = \iint dz\,dz' M'(z-z(a))\langle \mathbf{r}|L^{-1}|\mathbf{r}'\rangle M'(z'-z'(a')) \tag{3.8}$$

DH stands for drumhead. The main driving force for interface motion here is its curvature. Then we find

$$H \to H_{DH} = \sigma A = \sigma\int da \tag{3.9}$$

where $\sigma \equiv \int dx M'(x)^2$ is the surface tension and A is the total interfacial area. The interface equation of motion is now

$$-\frac{\delta H_{DH}}{\delta z(a)} = \frac{\delta F_{DH}}{\delta v(a)} - \frac{\delta \Delta F_{DH}}{\delta \theta(a)}\,. \tag{3.10}$$

Noting that $-\delta A/\delta z(a)=K(a)$ is the mean curvature of the interface (3.10) becomes

$$v(a) = \sigma\int\langle a|\Gamma|a'\rangle K(a')\,da' + \theta(a), \tag{3.11}$$

The fluctuation-dissipation relation is

$$\langle \theta(at)\theta(a't')\rangle = 2k_B T\langle a|\Gamma|a'\rangle\delta(t-t'). \tag{3.12}$$

In particular, when L is a constant, we have $\langle a|\Gamma|a'\rangle=\sigma^{-1}\delta(a-a')$ and [6]

$$v(a) = LK(a) + \theta(a) \tag{3.13}$$

As in the one-dimensional case, (3.11) or (3.13) can be made the basis for studying kinetics of fluctuations in systems quenched into thermodynamically unstable states. Very recently (3.13) without $\theta(a)$ was used to compute the scaling function of the non-equilibrium order parameter pair correlation [4]. Basic idea is to map $S(\mathbf{r},t)$ with sharp variations near interfaces into another smooth function $u(\mathbf{r},t)$ which vanishes on the interfaces. Statistical properties of $\{S\}$ is obtained by knowing that of $\{u\}$, and the latter can be studied in the Gaussian approximation. The results are in impressive agreements with the computer simulations of two and three dimensional kinetic Ising models [5]. A part of the success, we feel, may be ascribed to the mapping from S to u which can be viewed as a mapping of a strongly nonlinear problem to a weakly nonlinear one. One surpring fact in this connection we have uncovered recently is the asymptotical equivalence of pair correlations that are found here and in the weak coupling approach [7]. However, it is not so surprising after all if we note that discarding certain gradient terms in [7] can lead to formations of infinitely sharp phase boundaries in the long time regime. The pair correlation obtained is $(2/\pi)\sin^{-1}\exp(-x^2/4)$ with x a scaled distance.

The present interface model has been extended to conserved systems [8] and to binary critical fluids [9].

4. Vortex Lines

In this section we apply our general approach to the Pitaevskii model of superfluid Helium which takes the form [10]

$$\dot{\psi}(r,t) = -L\frac{\delta H(t)}{\delta\psi^*(r,t)} + f_\psi(r,t) \tag{4.1}$$

where $L=L_1+i$, $(L_1>0)$, with the atomic mass of Helium chosen to be unity and ψ is the complex order parameter. The coarse-grained free energy is

$$H\{\psi\} = \frac{1}{2}\int dr\left[\frac{1}{2}|\nabla\psi|^2 + g|\psi|^2\left(\frac{1}{2}|\psi|^2 - 1\right)\right] \tag{4.2}$$

$f_\psi(\mathbf{r},t)$ is the random force satisfying

$$\langle f_\psi^*(r,t)f_\psi(r',t')\rangle = 2L_1k_BT\delta(r-r')\delta(t-t'), \tag{4.3}$$

For our purpose it is useful to make explicit the fact that the complex equations (4.1) has in fact two independent components by introducing

$$\Psi = \begin{pmatrix}\psi\\ \psi^*\end{pmatrix}, \quad \zeta = \begin{pmatrix} f_\psi \\ f_\psi^*\end{pmatrix}, \quad \Gamma = \begin{pmatrix} L_1+i & 0\\ 0 & L_1-i\end{pmatrix} \tag{4.4}$$

(4.1) then becomes

$$\dot{\Psi} = -\Gamma\cdot\frac{\delta H}{\delta\Psi^\dagger} + \zeta \tag{4.5}$$

where a dagger denotes a Hermitian conjugate and H is now expressed in terms of $\Psi^\dagger$ and Ψ. (4.5) can be cast into the following variational form

$$-\frac{\delta H}{\delta\Psi^\dagger} = \frac{\delta F}{\delta\dot{\bar{\Psi}}^\dagger} - \frac{\delta\Delta F}{\delta\bar{\zeta}^\dagger} \tag{4.6}$$

with

$$F\{\dot{\bar{\Psi}}\,\dot{\Psi}\} \equiv \int dr\,\dot{\bar{\Psi}}^\dagger(r,t)\cdot\Gamma^{-1}\cdot\dot{\Psi}(r,t) \tag{4.7}$$

and $\Delta F \equiv F\{\bar{\zeta},\zeta\}$.

Here, nonuniform stationary states Ψ_0 or ψ_0 are those with a single straight vortex line which satisfy

$$\frac{\delta H}{\delta\Psi_0^\dagger} = \frac{\delta H}{\delta\psi_0^*} = \left[-\frac{1}{2}\nabla^2 + g(|\psi_0|^2 - 1)\right]\psi_0 = 0, \tag{4.8}$$

Other states Ψ with different vortex line configurations are no longer stationary. Denoting positions of vortex lines by $\mathbf{r}(\tau)$ and its velocity by $\mathbf{v}_\ell(\tau)$ where τ is the parameter along vortex lines, we make the ansatz:

$$\dot{\Psi} = -v_\ell(\tau)\cdot\nabla\Psi, \quad \dot{\bar{\Psi}} = -\bar{v}_\ell(\tau)\cdot\nabla\Psi \tag{4.9}$$

and similarly for ζ and $\bar{\zeta}$. Then we find

$$F \to F_{VL}\{\bar{v}_\ell, v_\ell\} \equiv \frac{2\pi}{|L|^2}\int d\tau\left[L_1E\,\bar{v}_\ell(\tau)\cdot v_\ell(\tau) + \hat{\kappa}\cdot\bar{v}_\ell(\tau)\times v_\ell(\tau)\right] \tag{4.10}$$

and similarly for ΔF_{VL} where $\bar{\mathbf{v}}_\ell$ and $\mathbf{V}_\ell$ are replaced by $\bar{\boldsymbol{\theta}}_\ell$ and $\boldsymbol{\theta}_\ell$, respectively, which are the random forces acting on vortex lines, where $\hat{\kappa}$ is the unit vector along a vortex line and $E \simeq E_0 + \ln R/r_0$. Here E_0 is some finite number, and R and r_0 are some characteristic length of vortex configuration and vortex line core radius, respectively [11]. H now reduces to the free energy of vortex lines H_{VL} given by

$$H_{VL} = \frac{\kappa}{4} \iint d\tau d\tau' \frac{1}{|\mathbf{r}_\ell(\tau) - \mathbf{r}_\ell(\tau')|} \mathbf{r}_\ell'(\tau) \cdot \mathbf{r}_\ell'(\tau') \tag{4.11}$$

where κ is the circulation of a quantized vortex line and $\mathbf{r}_\ell'(\tau) \equiv d\mathbf{r}_\ell(\tau)/d\tau$. The stochastic vortex line equation of motion is now

$$-\frac{\delta H_{VL}}{\delta \mathbf{r}_\ell(\tau)} = \frac{\delta F_{VL}}{\delta \mathbf{V}_\ell(\tau)} - \frac{\delta \Delta F_{VL}}{\delta \boldsymbol{\theta}_\ell(\tau)} \tag{4.12}$$

or more explicitly,

$$\mathbf{v}_\ell - \mathbf{v}_{s\perp}(\mathbf{r}_\ell) = \frac{L_1}{|L|^2}(L_1 \mathbf{V}_\ell + E\, \hat{\mathbf{v}}_\ell \times \hat{\kappa}) + \frac{1}{|L|^2}(\boldsymbol{\theta}_\ell - L_1 E \boldsymbol{\theta}_\ell \times \hat{\kappa}) \tag{4.13}$$

where $\mathbf{v}_{s\perp}(\mathbf{r}_\ell)$ is the component of the superfluid velocity $\mathbf{v}_s(\mathbf{r}_\ell)$ at $\mathbf{r}_\ell$ perpendicular to the vortex line with

$$\mathbf{v}_s(\mathbf{r}) = -\frac{\kappa}{4\pi} \int d\tau\, \mathbf{r}_\ell'(\tau) \times \nabla \frac{1}{|\mathbf{r} - \mathbf{r}_\ell(\tau)|}. \tag{4.14}$$

Similar equation as (4.13) without random forces has been derived by Onuki by a different method [12].

We have not been able to use the vortex equation of motion derived here for phase transition problems although this kind of equation has been successfully applied to superfluid turbulence [13]. An interesting possibility is to consider the dynamics of rotating liquid ^{4}He suddenly quenched below the λ temperature [14].

5. Concluding Remarks

In the preceding sections we have obtained stochastic equations of motion for some representative types of singularities from the underlying nonlinear field equations. However, these nonlinear field equations themselves are often highly idealized models of real systems whereas these topological singularities have meanings going beyond these idealized continuum models. Hence there is a problem of understanding these topological singularities on the basis of more realistic possibly discrete models. On the other hand, we still face a formidable statistical problem of understanding macroscopic behavior on the basis of topological singularities [1, 15].

Appendix

Here we extend the method of Sec.1 to the following stochastic field equation with the inertial term added to (2.1):

$$m\ddot{y} + \dot{y} = -L(\partial_x)\frac{\delta H}{\delta y} + f. \tag{A.1}$$

Since the analysis closely parallels that of Sec.1 we only indicate new aspects arising from introduction of the inertia term. Here we must allow a single kink moving with a constant velocity v instead of a stationary kink described by (2.4). Thus we have for the i-th kink,

$$[m v_i^2 \partial_x^2 - v_i \partial_x] M_i'(x) = L(\partial_x)[\partial_x^2 - gV''(M_i(x))] M_i'(x). \tag{A.2}$$

Hence $M_i(x)$ also depends upon v_i. Then, besides H we also need a Lagrangian $\mathcal{L}$=K-H, K being a "kinetic energy" given by

$$K \equiv \frac{m}{2} \int \dot{y} L^{-1} \dot{y}\, dx \tag{A.3}$$

(A.1) now becomes with $\partial_t \equiv \partial/\partial t$

$$-\partial_t \frac{\delta \mathcal{L}}{\delta \dot{y}} + \frac{\delta \mathcal{L}}{\delta y} = \frac{\delta F}{\delta \dot{y}} - \frac{\delta \Delta F}{\delta f}. \tag{A.4}$$

Using (2.6) we find

$$\dot{y} = \sum_j (-\dot{x}_j M_j' + \dot{v}_j M_j^v) \tag{A.5}$$

where a superfix v denotes differentiation with respect to v. On the other hand f remains the same as in Sec.1. Therefore we find in terms of kink variables $\{x_i, v_i\}$,

$$K_K\{x v \dot{x} \dot{v}\} = \frac{m}{2}\sum_{ij}\left[(M_i', L^{-1}M_j')\dot{x}_i\dot{x}_j - 2(M_i', L^{-1}M_j^v)\dot{x}_i\dot{v}_j + (M_i^v, L^{-1}M_j^v)\dot{v}_i\dot{v}_j\right] \tag{A.6}$$

and similarly for $F_K = (1/m)K_K$. ΔF_K remains unchanged. H now reduces to $H_K\{x,v\}$.

In deriving the kink equations of motion from (A.4) we consider virtual infinitesimal variation of y due to infinitesimal variations of x_i where the variables $\{v_i\}$ are kept fixed treating $\dot{x}_i$ and v_i as though independent variables. Then the kink equation of motion is formally written as

$$-\partial_t \frac{\partial \mathcal{L}_K}{\partial \dot{x}_i} + \frac{\partial \mathcal{L}_K}{\partial x_i} = \frac{\partial F_K}{\partial \dot{x}_i} - \frac{\partial \Delta F_K}{\partial \theta_i} \tag{A.7}$$

Here v and $\dot{v}$ are held fixed in carrying out partial differentiations (∂t, however, acts on all the variables). Thus, for example, we have

$$-\partial_t \frac{\partial K_K}{\partial \dot{x}_i} + \frac{\partial K_K}{\partial x_i} = -m\sum_j\left[(M_i', L^{-1}M_j')\ddot{x}_j - (M_i', L^{-1}M_j'')\dot{x}_j^2 + 2(M_i', L^{-1}M_j'^{v})\dot{x}_j\dot{v}_j\right] \tag{A.8}$$

$$\frac{\partial F_K}{\partial \dot{x}_i} = \sum_j\left[(M_i', L^{-1}M_j')\dot{x}_j - (M_i', L^{-1}M_j^v)\dot{v}_j\right] \tag{A.9}$$

where we have ignored terms of $O(\dot{v}^2, \ddot{v})$ since time variations of $\{v_i\}$ are small. Next we turn to H_K, for which we have with (2.3)

$$\frac{\partial H_K}{\partial x_i} = -\int dx\, M_i'\left\{\partial_x^2(M_i + \delta y_i) + g\left[V'(M_i) + \delta y_i V''(M_i) + V_{nli}\right]\right\} \tag{A.10}$$

where we have split y of (2.7) as $M_i + \delta y_i$ and have introduced Vnli by

$$V_{nli}' = V'(M_i + \delta y_i) - V'(M_i) - \delta y_i V''(M_i) \tag{A.11}$$

Vnli starts out with second power of the small quantity δy_i. Integrating by parts and dropping terms which arise from boundaries we obtain using (A.2) and (2.7)

$$\frac{\partial H_K}{\partial x_i} = \sum_{j\neq i}\int dx\, M_i' L^{-1}\left[m(v_i^0)^2\partial_x^2 - v_i^0\partial_x\right]M_j(x) - g\int dx\, M_i'(x)V_{nli}'. \tag{A.12}$$

Substituting (A.8), (A.9), (A.12) into (A.7) and using

$$m(v_i^0)^2(M_i', L^{-1}M_i'') - v_i^0(M_i', L^{-1}M_i') = 0 \tag{A.13}$$

which follows from (A.2) we finally obtain

$$\sum_j\Big\{(M_i', L^{-1}M_j')\left[m\dot{v}_j + v_j - v_j^0 - \theta_j\right] - m(M_i', L^{-1}M_j'')\left[v_j^2 - (v_j^0)^2\right] + 2m(M_i', L^{-1}M_j'^{v})v_j\dot{v}_j - (M_i', L^{-1}M_j^v)\dot{v}_j\Big\} = g\int dx\, M_i' V_{nli}' \tag{A.14}$$

which is basically identical to that found in [2]. There the right hand side of (A.14) is also evaluated and is found to produce the right hand side of (2.11) for the special case considered.

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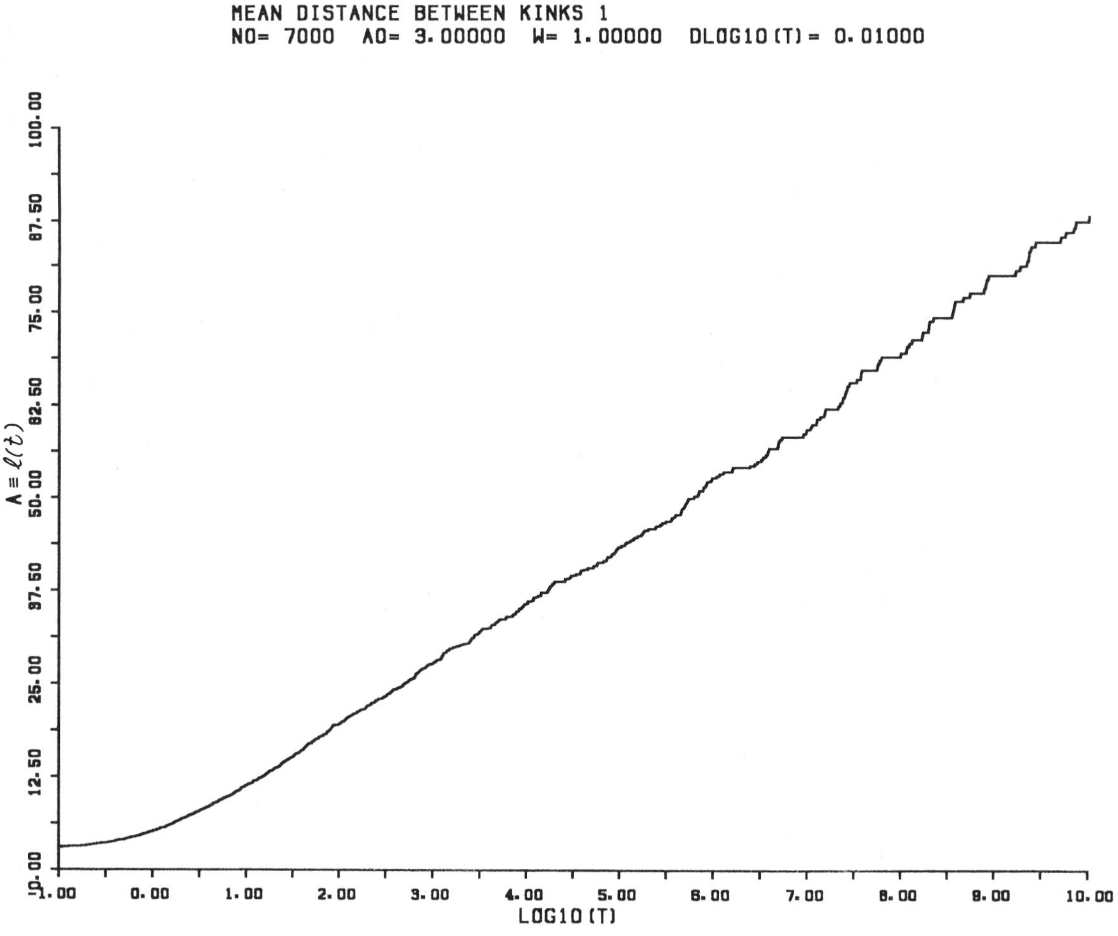
Note added:

We have numerically solved (2.13) with $f_i=0$ for the system of the length $\Lambda=2{,}1000$ times ξ to obtain the average domain size $\ell(t)=\Lambda/N(t)$, where $N(t)$ is the number of kinks and antikinks. The initial spatial distributions of kinks and antikinks $\{x_i(0)\}$ have been given by sets of random numbers. Whenever one kink and one antikink contact each other $N(t)$ decreases by two. We show in the figure on the next page the result for $N(0)=7000$, $\ell(0)=3$, $\xi=1$, which well supports the qualitative argument of Sec.2 predicting $\ell(t)\propto\ln t$.


MEAN DISTANCE BETWEEN KINKS 1
NO= 7000 AO= 3.00000 W= 1.00000 DLOG10(T)= 0.01000
$\Lambda \equiv \ell(t)$
0.00
12.50
25.00
37.50
50.00
62.50
75.00
87.50
100.00
-1.00
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
LOG10(T)