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COHERENT STRUCTURE FORMATION OF VORTEX FLOW AROUND A SINK

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Measurements of the spatial distribution of the angular momentum for a vortex generated around the sink reveal that the spatially coherent vortex is organized from random streams produced on the external boundary. The two-dimensional flow simulation based on the Navier-Stokes equation has been carried out for comparison with the experiment.

1. Introduction

A fluid flowing centripetally toward a sink in the bottom of a vessel has a tendency to form a spiral vortex around it. Large scale of vortices of this kind in meteorology are tornadoes, cyclones and typhoons which are produced by a constant supply of energy from strong local convection. The formation of vortex is maintained by providing a constant energy, so that it is regarded as an example of dissipative structure presented by Prigogine [1]. The stability of the dissipative structure of vortex depends on the amount of flow Q flown out per unit time through the sink; for small values of Q the vortex is not generated or unstable temporally and spatially even if it is generated, while for large values of Q, a large scale coherent structure of vortex flow is generated [2,3].

When we consider quantitatively the spatial ordering of a dissipative structure, it is desirable to know the spatial mean value and fluctuation of a dissipative quantity concerned. Thus, in order to check the spatial order of vortex, we have measured the two dimensional distribution of the flow velocity on the surface of glycerin-water solution, by means of the floating method using Al powder as a tracer and obtained the mean value of the azimuthal velocity and its fluctuation on circles of various radii with the centre at the sink. For comparison with experimental results, a computer simulation based on the Navier-Stokes equation expressed in terms of polar coordinates has been carried out.

2. Experimentals

We used two vessels for this experiment; one is in the shape of a rectangle and the other has a dodecagon shape. In the case of rectangular vessel (120 cm×30 cm), the flow is flown out through a orifice (8 mmϕ) in the centre of the bottom and the same amount of flow is supplied through two intakes located at both ends of the rectangle. Examples of streamlines of flow for three values of the amount of discharge Q are shown in Fig. 1. For small value of Q, many small vortices appear but their distribution is very unstable. For Q=137 cc/sec, we can see four vortices around the sink; a diagonal pair of the top-right and the bottom-left turns clockwise and another pair of the top-left and the bottom-right turns counterclockwise. With increasing amount of discharge, the superior pair overcomes the other pair and the four vortices are unified into a single vortex. Then, the velocity of vortex flow increases and the flow pattern becomes coaxial circles about the sink.

In order to obtain an isotropic inflow, we made another vessel of the dodecagon shape as shown in Fig. 2. Each of twelve intakes attached to each side of the dodecagon feeds an almost equal amount of flow and the total amount of flow is flown out through the sink. Photograghs of streamlines of flow with this dodecagon shape vessel is shown in Fig. 3. In this case, as Q increases, a single vortex is found to occur immediately following an unstable flow. We have measured the azimuthal velocity v_θ from the length of each trace within a circle of radius 12 cm around the sink in these photograghs and obtained its distribution as a function of distance r from the centre of the sink. Further we have calculated the distribution of the angular momentum $L=rv_\theta$.

Results of v_θ and L are shown in Fig. 4. Note that the scattering of points represents the spatial fluctuation and not the temporal fluctuation. In these cases where a single vortex is developed, in the range of r<12 cm the mean value of v_θ is nearly inversely proportional to r, thus the angular momentum is nearly conserved. The mean values of v_θ and L increase as Q increases. As for the fluctuation of the angular momentum, it decreases with decreasing r; this means that the spatial coherence of vortex gets better when the flow advances toward the central sink.

It is noticeable that for large value of Q, the spatial fluctuations Δv_θ and ΔL are enhanced and the relation between Δv_θ and Q was found to be given by

$$\langle \Delta v_\theta{}^2 \rangle_{r=\mathrm{const}} \propto Q^2 \qquad (1)$$

from a series of data. In general, in hydrodynamical systems, a supply of flow into the system is thought to be accompanied by flow velocity fluctuation, since a flow randomization at the inlet of the system is brought about by a dismatching of the flow impedance between the outside and the inside of the system. The variance of fluctuation in such an open system may be proportional to the square of the mean flow, that is, the power supplied to the system.

3. Computer Simulation

The vortex flow around a sink has a three-dimensional structure; v_θ takes a large value near the surface of the fluid, while it is restrained near the bottom, and the z component of flow also arises around the sink. Using the two dimensional flow approximation, however, we have tried to simulate the behavior of vortex which we saw in the preceding section. We start with the Navier-Stokes equations expressed in terms of polar coordinates [4],

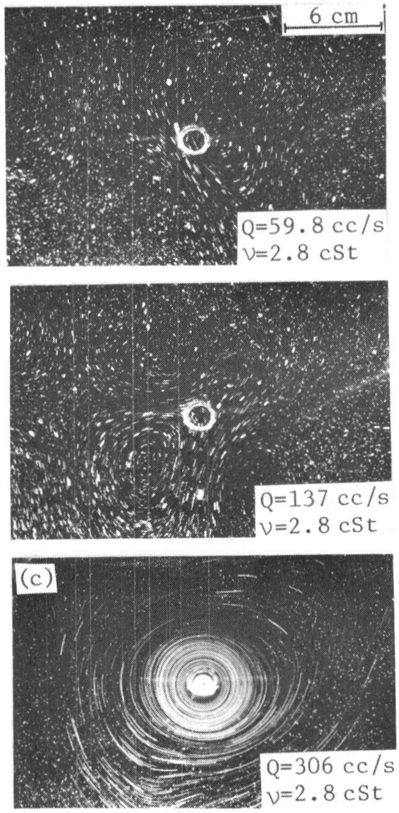

Fig. 1. Streamlines around the sink in the rectangular vessel. Q is the amount of discharge flow through the sink and ν the kinematic viscosity.

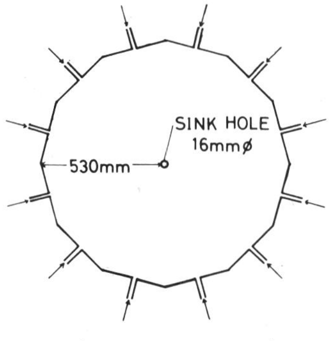

Fig. 2. Dodecagon shape vessel.

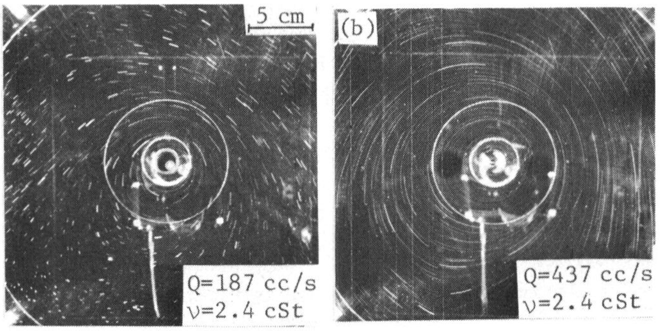

Fig. 3. Streamlines around the sink in the dodecagon shape vessel.

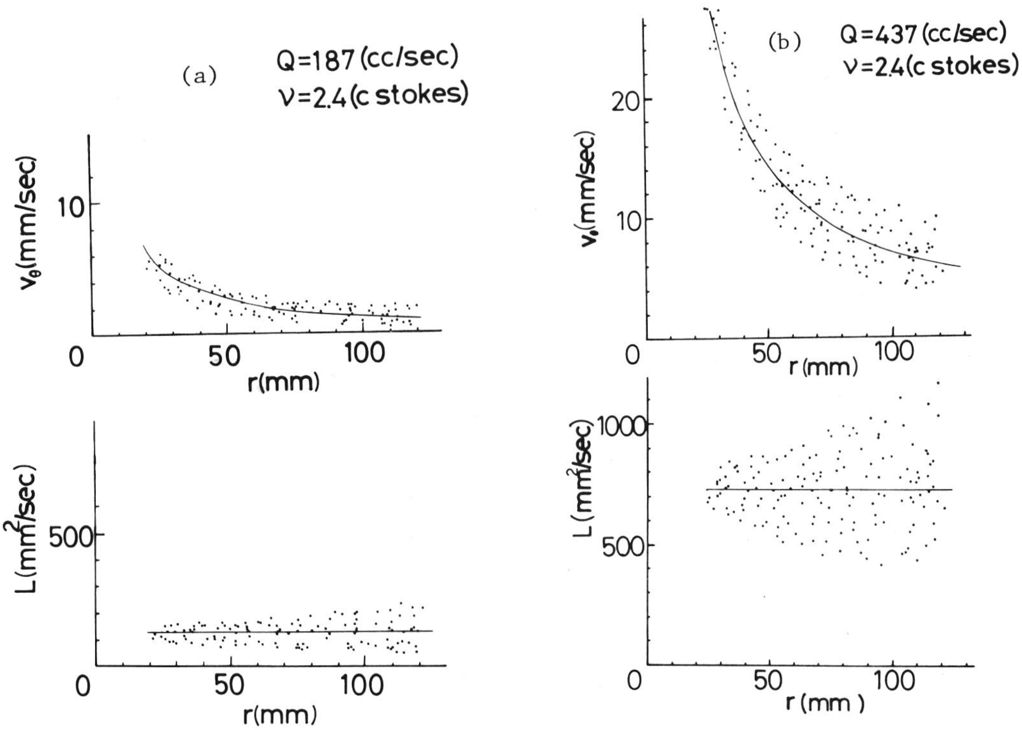

Fig. 4. The distribution of the azimuthal velocity v_θ and the angular momentum $L=rv_\theta$ in the dodecagon shape vessel.

$$\frac{\partial V_r}{\partial t} = -V_r\frac{\partial V_r}{\partial r} - \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + \frac{V_\theta^2}{r} - \frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{V_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right) \tag{2}$$

$$\frac{\partial V_\theta}{\partial t} = -V_r\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} - \frac{V_r V_\theta}{r} + \nu\left(\frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right) \tag{3}$$

and the continuity equation

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} = 0, \tag{4}$$

where ρ is the density, p the pressure and ν the kinematic viscosity. Here we divide the radial velocity V_r into the mean value $-Q/2\pi r$ and the perturbation v_r, and for V_θ we take only the perturbation v_θ, that is,

$$V_r = -Q/2\pi r + v_r \tag{5}$$

$$V_\theta = v_\theta \tag{6}$$

By substituting (5) and (6) into (2) and (3) and taking account of the continuity relation (4), we get the following equation for the perturbations v_r and v_θ,

$$\frac{\partial v_r}{\partial t} = \left[-\frac{Q}{\pi}v_r - \left(\frac{Q}{2\pi} + \nu\right)\frac{\partial v_\theta}{\partial \theta} + \nu\frac{\partial^2 v_r}{\partial \theta^2}\right]\frac{1}{r^2} + \left[v_r^2 + v_r\frac{\partial v_\theta}{\partial \theta} - v_\theta\frac{\partial v_r}{\partial \theta} + v_\theta^2 - \nu\frac{\partial^2 v_\theta}{\partial r\partial \theta}\right]\frac{1}{r} \tag{7}$$

$$\frac{\partial v_\theta}{\partial t} = \left[\left(\frac{Q}{2\pi} - \nu\right)v_\theta + \nu\left(\frac{\partial^2 v_\theta}{\partial \theta^2} + 2\frac{\partial v_r}{\partial \theta}\right)\right]\frac{1}{r^2}$$

$$+ \left[\left(\frac{Q}{2\pi} + \nu\right)\frac{\partial v_\theta}{\partial r} - v_r v_\theta - v_\theta\frac{\partial v_\theta}{\partial \theta}\right]\frac{1}{r} + \nu\frac{\partial^2 v_\theta}{\partial r^2} - v_r\frac{\partial v_\theta}{\partial r}. \tag{8}$$

We impose a spatially random but time invariant boundary codition for v_θ and a zero value condition for v_r on the external boundary circle. According to the empirical law (1), we took the random values of v_θ on the boundary to be proportional to the amount of discharge Q. A typical example of steady state flow pattern in case of $Q/2\pi=2.0$ and $\nu=0.2$ is represented in Fig. 5. Both of the clockwise and counterclockwise streams are given on the external boundary circle, but on advancing toward the central sink, the counterclockwise stream which is accidentally superior to the other on the boundary predominates and develops into a coherent vortex near the sink.

The distributions of v_θ and L are shown in Fig. 6. A comparison of the r dependences of the mean value in Fig. 6(a) and (b), indicates that the conservation of angular momentum is not fully satisfied for small value of Q and it becomes more perfect for large Q. The spatial fluctuation of the angular momentum produced on the boundary circle is found to be smaller as the flow advances toward the central sink; this is in qualitative agreement with experimental results shown in Fig. 4. It should be noticed, however, that when Q is kept constant an organization of coherent vortex is more remarkable for large value of kinematic viscosity ν.

Though the present computer simulation is based on the two-dimensional flow approximation, it has qualitatively accounted for the formation of coherent vortex from the random boundary flow. An extension to the three-dimensional flow model would be desirable for a more complete agreement.

References

[1] G. Nicolis and I. Prigogine: Selforganization in nonequilibrium systems (Wiley, 1977).

[2] T. Kawakubo, Y. Tsuchiya, M. Sugaya and K. Matsumura: Phys. Lett. 68(1978)65.

[3] T. Kawakubo, S. Kabashima and Y. Tsuchiya: Prog. Theor. Phys. Suppl. 64(1978)150.

[4] L.D. Landau and E.M. Lifshitz: Fluid Mechanics (Pergamon, London, 1959).

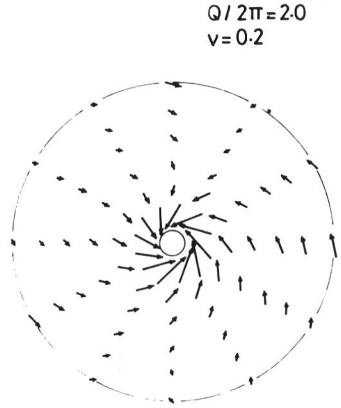

Fig. 5. Computer simulation of the coherent vortex formation when a random flow is given on the external boundary circle.

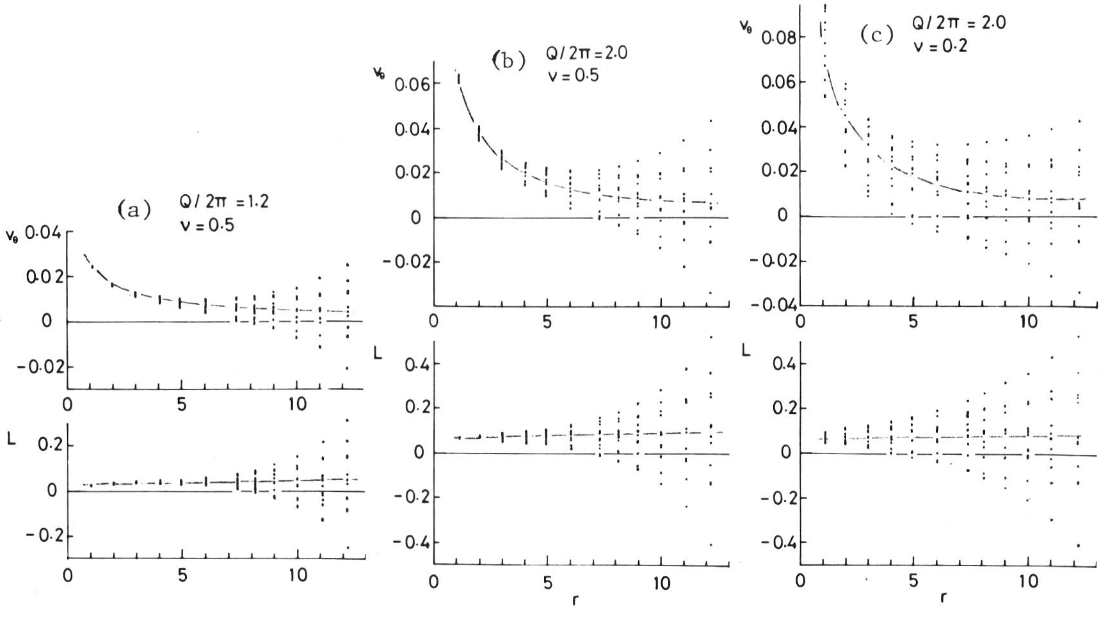

Fig. 6. Computer simulation of the distribution of v_θ and L.