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CHAOTIC STATES IN DRIVEN FERROMAGNETS

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A study on nonequilibrium states of driven ferromagnets is developed. A global stability analysis of fixed lines (i.e., steady states) and the associated variation of the stable attractors are presented, by changing the strength of the interaction between "Cooper pairs" of spin waves. The puzzling wide-spread spectrum of driven modes pointed out by Gottlieb and Suhl and by Anderson is now attributed to the onset of chaotic behaviors of these modes. The Feigenbaum's scaling constant and the Poincaré section beyond the accumulation point are also given.

Parametric amplification phenomenon of spin waves in driven ferromagnets is pointed out to be a typical example of "broken symmetry in dissipative structures" [1]. In our preliminary report, we have studied nonequilibrium states of ferromagnets under a strong parallel pumping field beyond the Suhl threshold[2]. In contrast with the situation encountered in the broken symmetry in equilibrium states, the condensation involving not just a single isolated mode but two (or, more precisely, finite number of) modes with wave vectors confined to the resonance surface has been found to be operative so as to cause either periodic or nonperiodic temporal variations in the intensities of these modes[2]. This phenomenon resembles that of rippled patterns in sand on the seashore with their directions showing periodic or nonperiodic temporal variations according to unsteady directions of the wind[1].

On the other hand, a stochastic temporal behavior of the magnetization in the driven YIG has received growing experimental interests among Russian groups [3],[4], although neither a strange attractor nor the route towards it has not to date been obtained. It is therefore highly desirable to develop further our previous study.

In this paper, we present a global stability analysis of fixed lines and the associated variation of the stable attractor, by changing the strength of the interaction between "Cooper pairs" of spin waves. We then calculate the Feigenbaum's constant δ in the period-doubling region and show the Poincaré sections of the strange attractor beyond the accumulation point. As before[2], we employ the symmetric two-modes model, assuming the presence of predominant humps in the state density around two different wave vectors within the resonance surface. In the post-threshold region, dynamics of parametrically excited spin-waves pairs is described by the equations for real and imaginary parts of "Cooper pair" densities σ_j $(j=1,2)$:

$$\frac{1}{2}\frac{d}{dt}m_j = -\gamma m_j + [\,\Delta\omega + (S+T)(m_j^2+\ell_j^2)^{\frac{1}{2}} + T'(m_{j'}^2+\ell_{j'}^2)^{\frac{1}{2}}\,]\,\ell_j + S'(m_j^2+\ell_j^2)^{\frac{1}{2}}\,\ell_j \quad , \qquad (1a)$$

$$\frac{1}{2}\frac{d}{dt}\ell_j = -\gamma \ell_j - [\,\Delta\omega + (S+T)(m_j^2+\ell_j^2)^{\frac{1}{2}} + T'(m_{j'}^2+\ell_{j'}^2)^{\frac{1}{2}}\,]\,m_j$$

$$+ (hV + S'm_{j'})(m_j^2 + \ell_j^2)^{\frac{1}{2}} , \tag{1b}$$

where $m_j = \mathrm{Re}\,\sigma_j$, $\ell_j = \mathrm{Im}\,\sigma_j$. Here we have rewritten the equation of motion in [2],[3], noting the asymptotic conservation law $n_j^2 = m_j^2 + \ell_j^2$ where n_j denotes the intensity (population) of the j-th mode. In equations (1a) and (1b), (j,j') is (1,2) or (2,1). γ and $\Delta\omega$ are the dissipation rate and the off-resonance frequency between the spin-wave energy and one-half of the frequency of the external field with magnitude hV , respectively. S,T and S',T' are the diagonal and nondiagonal interactions between "Cooper pairs", respectively; S' retaines the phases of these pairs, while T' destroys them. A set of equations (1a) and (1b) are the four-dimensional autonomous equations. Their stable fixed points are the states where collective excitations reminiscent of second sound of spin waves are expected to appear.

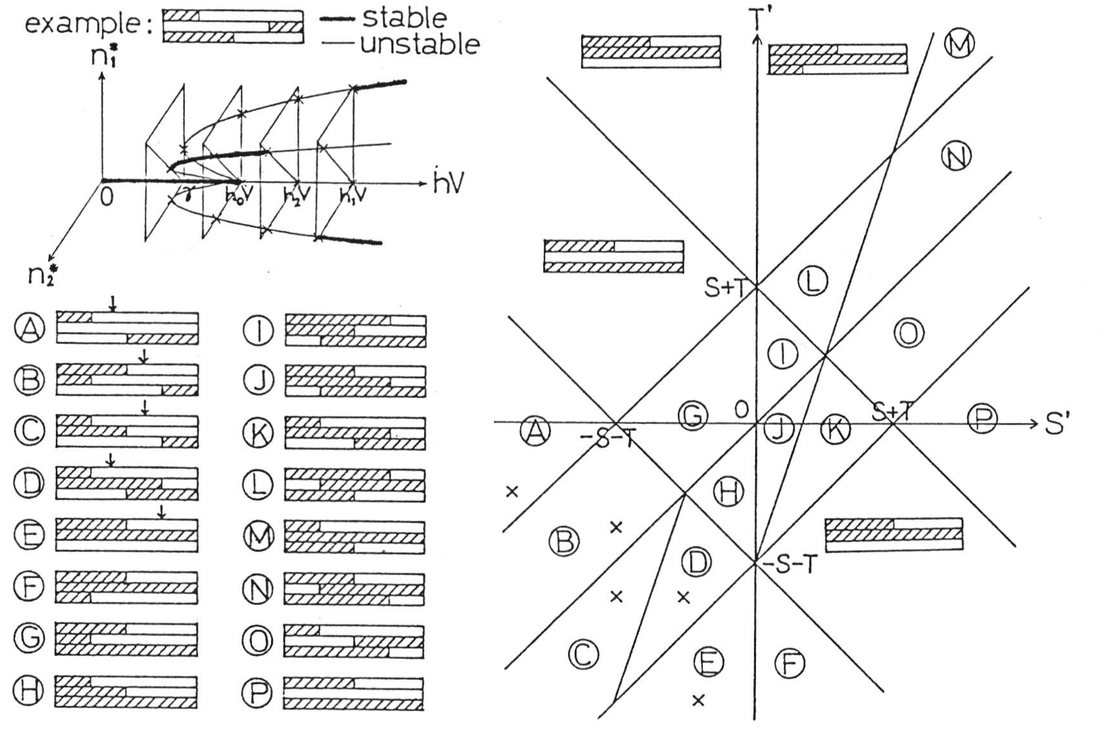

Figure 1. Global stability diagram of fixed lines. Each rectangle has 3 rows: The top is a trivial fixed line in the text, while the middle and bottom being the asymmetric and symmetric ones, respectively. Their abscissa denotes hV . The hatched part is stable, while the remaining one being unstable. The transition from a stable part to an unstable one (or vice versa) occurs at h_0V , h_1V and h_2V for fixed lines associated with the top, middle and bottom rows, respectively; $h_0V = (\gamma^2 + \Delta\omega^2)^{1/2}$, $h_1V = [\gamma^2 + \{ \Delta\omega(S+T+S'-T')/(S+T-S'+T') \}^2]^{1/2}$, $h_2V = [\gamma^2 + \{ 2\Delta\omega S'/(S+T-S'+T') \}^2]^{1/2}$.

For continuously varying values of hV, we obtain a sequence of fixed points, i.e., fixed lines[2]. Besides the trivial fixed line ($n_1 = n_2 = 0$), one symmetric ($n_1 = n_2 \neq 0$) and two asymmetric ($n_1 \neq 0$, $n_2 = 0$ and vice versa) fixed lines have been provided [2] in the physically relevant region (n_1 , $n_2 \geq 0$) for particular values of S' and T'. Here we show in figure 1 a global stability diagram of these lines for various values of S' and T' under the condition S+T>0 and $\Delta\omega$<0. In contrast with well-studied dissipative systems(e.g. Lorenz system), the present system always involves more than one stable fixed points for any value of S' ,T' and hV. Despite this fact, the computed temporal evolution of equations (1a,b) shows the presence of the periodic or nonperiodic(chaotic) stable attractor coexisting

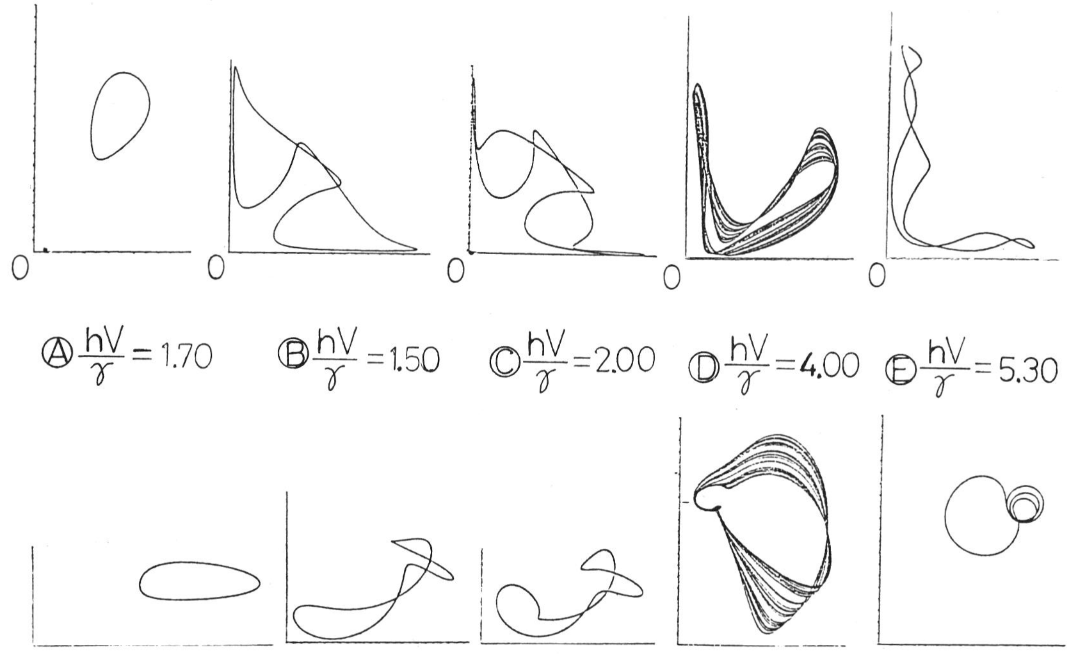

Figure 2. Variations of periodic or chaotic attractor. The upper and lower parts are its projections onto $n_1 - n_2$ and $m_1 - \ell_1$ planes, respectively. In each attractor, the values S' and T' employed are given by marks × and the value of hV is indicated by the arrow ↓ on the rectangle in figure 1.

with these points (see figure 2), except for the region where the symmetric fixed line is stable. Figure 2 indicates the strong sensitivity of the attractor to the values of S' and T'. The interesting variation of the attractor can be expected to occur in the vicinity of some of the boundary region in figure 1 where fixed points exhibit marginal stability, which will be described elsewhere. Figure 3 shows in the period-doubling region the external-field dependence of the value m_1 for the set of points on the Poincaré section of the plane $\ell_1 = 0$ with $m_1 > 0$. It shows a typical bifurcation diagram, from which we estimate the Feigenbaum's constant as $\delta = 4.675$ and the accumulation point as $h_\infty V/\gamma = 5.571$; The previous value for the latter [2] has been a bit improved here by the present high-precision computation. The final figure shows the Poincare section of the attractor for increasing values of hV beyond $h_\infty V$ for the same values S' and T' as used in figure 3. We find that a band-merging transition leads to a developed strange attractor. A careful examination of the Poincaré section reveals the presence of the stretching and folding mechanism operating during constructing the strange attractor. This mechanism makes the dimension of the attractor fractal. Further increase of the magnitude hV can induce a hyperchaotic behavior, whose structure as well as an associated Lyapunov spectrum will be described elsewhere. Finally it should be mentioned that the puzzling wide-spread spectrum of the driven modes pointed out in [1] and [5] is reasonably attributed to the onset of chaotic behaviors described in the present article.

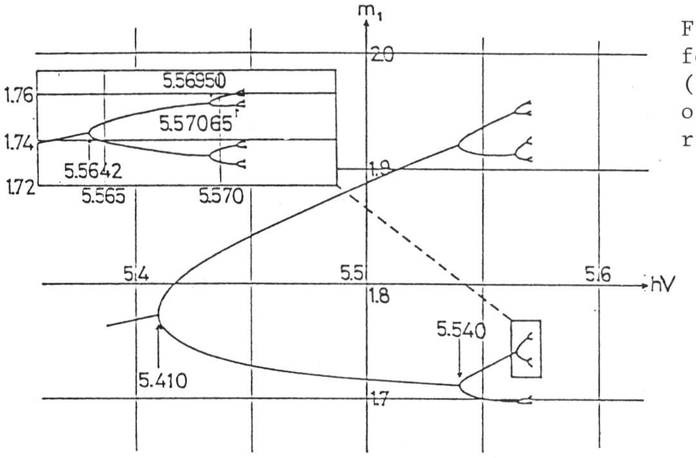

Figure 3. Bifurcation diagram for $S'/\gamma = -0.8$, $T'/\gamma = -3.0$ $(S+T)/\gamma = 2.0$. Abscissa and ordinates are hV/γ and m_1 , respectively.

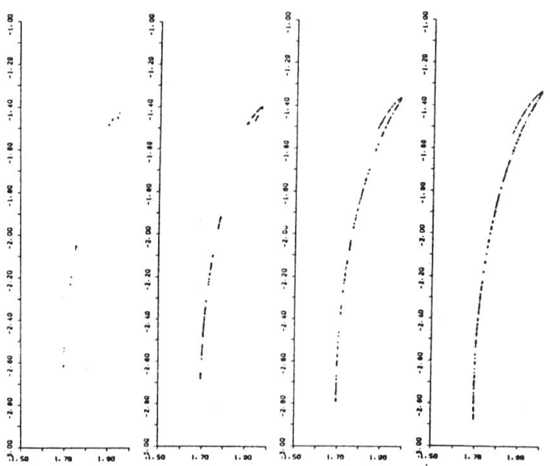

Figure 4. Poincaré section beyond $h_\infty V$. S' and T' are the same as in figure 3. From the left, $hV/\gamma = 5.57$(Limit cycle of the period 16), 5.58 , 5.59 and 5.60 .

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