J. Phys. Soc. Jpn. **52** (1983) Suppl. p. 167–170

A MAZE-LIKE PATTERN IN A MONODISPERSIVE LATEX SYSTEM AND THE FRUSTRATION PROBLEM

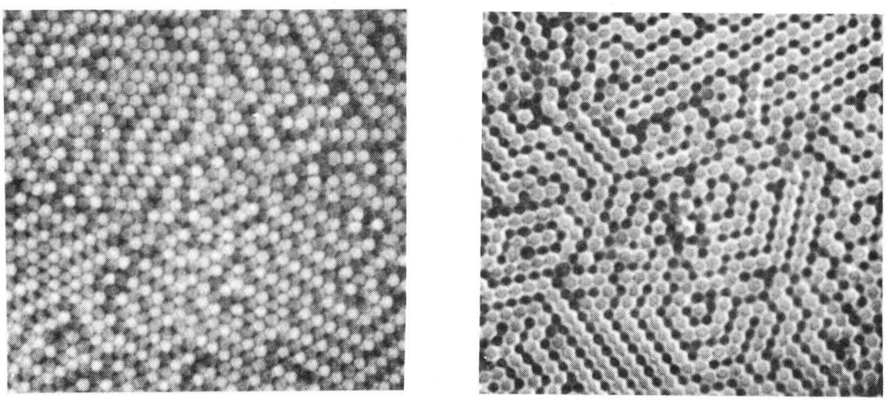
Tohru Ogawa

Institute of Applied Physics, University of Tsukuba,
Sakura, Ibaraki 305, Japan

The configurations of the degenerated ground states in an antiferromagnetic Ising model on a two-dimensional triangular lattice are morphologically studied. The motivation is twofold; One is to analyze the maze-like patterns observed in the latex system and the other is to give a basic discussion on the amorphous states from the viewpoint of statistical physics. The circumstances of a site in a state is classified into five classes by the part in a cluster.

1. Introduction

A maze-like pattern is observed in a monodispersive latex system in a thin gap between two glass plates by Koshikiya and Hachisu[1]. The diameter of latex particles is about 7000A. The electrolyte used is 10^{-5}mol/l KCl. The pattern is static in some cases and is moving in the other cases. In a sufficiently thin gap ($\sim 2\mu$), the latex particles form a two-dimensional triangular lattice in a layer. In a properly thick gap, their arrangement is uneven in a layer and the pattern appears as shown in Fig.1. In Fig.1, (i) and (ii) are thinner and thicker cases respectively. In a too thick gap ($\sim 3\mu$),



(i) (ii)

Fig.1

they form two layers without pattern. Each particle seems to lie in either of two states, upper or lower, at its lattice site. Two neighbouring particles prefer different states with each other so that they might have bigger free volumes. Then the system is analogous to the antiferromagnetic Ising model on a triangular lattice, which is a typical frustrated system. The ground state of this model is degenerated and the residual entropy at absolute zero is rigolously estimated by Wannier[2].

In this paper, the degenerated ground state is morphologically studied for two purposes: One is to analyze the above mentioned maze-like pattern in the latex system and the other is to discuss the amorphous states as a basic problem of statistical physics.

2. A model

Let us take an antiferromagnetic Ising model on a triangular lattice. Suppose the number of lattice sites is L and then that of nearest neighbour pairs is 3L. A basic cell is defined as a regular triangle consisting of three sites which are nearest neighbour with one another. The number of such basic cells is 2L. Let us confine our consideration to on a set of configurations which belong to the degenerated ground states. It may be referred to as the ground state ensemble. In any member of this ensemble, as easily seen, three sites in a basic cell never be in the same state; two of three sites are in one state and the other site in the other state. This constraint, which is hereafter referred to as the basic constraint, characterises the ensemble. A cluster is defined as the set of the connected sites which are in the same state. As for a site in one state, the configuration of its six neighbours is restricted to either of the five types

# # # O # # #	# # # O O # #	# # O O O # #	# O O O # # #	# O O O # # O
(i)	(t)	(s)	(a)	(b)
isolated	terminal	straight	angular	branching
孤	端	直	折	岐
O O O # O O O	O O O # # O O	O O # # # O O	O # # # O O O	O # # # O O #

Fig.2

shown in Fig.2. The classification is based on the part of the central site in a cluster consisting of the sites in the same state. It directly follows that the pattern of the whole system is maze-like in most cases. It sometimes takes place that the cluster of one state constructs a network like a honeycomb lattice and those of the other state are isolated, distributing as in a triangular lattice. The probability of five types of configurations in the ground state ensemble are calculated by making use of the Kikuchi's cluster variation scheme [3] , which is the systematic generalization of the Bethe approximation. A hexagon consisting of seven sites, one at the centre and six at the corners as in Fig.2, is chosen as the basic figure. The number of allowed configurations on it is ten, twice of the above mentioned five in view of two possibilities at the centre. If the equivalence of two states, upper and lower, is assumed, the number of configurations is essentially five. Then the number of independent variables is finally reduced to two. The basic equation of the cluster variation is easily solved and the probability of five configurations in Fig.2 is given as follows,

(i) 0.0168, (t) 0.0180, (s) 0.0194, (a) 0.0292 and (b) 0.0709,

where the symmetry factors by the existence of the equivalent configurations in various directions are not taken into account. If they are included, the probabilities are rewritten as

(i) 0.0336, (t) 0.2165, (s) 0.1163, (a) 0.3500 and (b) 0.2836.

The entropy in this approximation is 0.3154. It should be compared with the rigolous value 0.3231 obtained by Wannier [2] and corrected by Domb [4].

3. The pattern observed in the latex system

Now, the comparison of the results of the approximation theory in the last section with the pattern in the latex system is only preliminary and qualitative. The followings, however, may be pointed out.

(1) It is not sure whether the system is symmetrical with respect to the exchange of two states, upper and lower. In the terminology of magnetism, an external magnetic field may be exerted.

(2) The straight pattern seems to exist rather much in the latex system. Some other interactions than the nearest neighbour one may additionally exist. It should be noted that the model in the last section is essentially applicable even in such cases. What is important is that three sites in any basic cell never be or are very rarely in the same state. It is of no use to extend the phase space beyond the ground state ensemble. The effect of the additional interactions is to give the weight to the members of the ensemble.

(3) It is not clear now what kind of part is easy to move in a moving pattern. It is necessary to analyze the static pattern in terms of the model in the last section before advancing to a moving pattern, since it is impossible to recognize the moving part without establishing the concepts and a viewpoint on the static patterns.

4. Statistical physics of amorphous states

Suppose some mechanism of changing the configuration is assumed. For example, the following two mechanism are considered.

(1) the state of a single site can change if it does not violate the basic constraint. Only the central sites in the configuration (b) can change the state. Sometimes, changing at a site alters the circumstance of some neighbouring sites into type (b) and changing can propagate as shown in Fig.3.

# 0 # 0		# 0 # 0		# 0 # 0		# 0 # 0
0 $\underline{0}$ # 0 #	$\leftrightarrow$	0 $\underline{\#}$ $\underline{\#}$ 0 #	$\leftrightarrow$	0 # $\underline{0}$ $\underline{0}$ #	$\leftrightarrow$	0 # 0 $\underline{\#}$ #
# 0 # 0		# 0 # 0		# 0 # 0		# 0 # 0

Fig.3

An underlined site in Fig.3 can change its state without violating the basic constraint. This mechanism does not conserve the number of the sites in each state, which corresponds to the number of particles in the lattice-gas interpretation.

(2) The states in a nearest neighbour pair are exchanged if they do not violate the basic constraint. This mechanism conserves the number of sites in each state and may be suitable as a lattice gas model of amorphous. Changing states in this mechanism takes place only when one of the neighbouring two

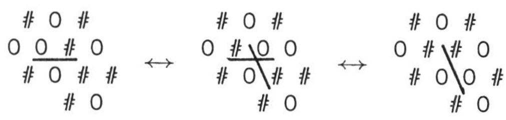


Fig.4

sites is in the configuration (a) and the other in (b). (See Fig.4) Two sites in an underlined pair in Fig.4 can exchange their states without violating the basic constraint.

It is noted that the mobility depends on the numbers of sites in each states especially in the mechanism (2).

If the number of sites in one state is 2L/3 and that of the other L/3, all the sites in the former state are in the configuration (b) and those in the latter in (i). It corresponds to a crystal and no motions are allowed in the mechanism (2).

If some sites in the major state is changed into the minor one, some sites can change its state with the neighbour but the motion will not extend to the remote sites. Then the configurations of the whole system are divided into many families: The configurations in the same family are connected by the allowed mechanism and those belonging to different families are disconnected. Such a configuration corresponds to an amprphous state.

If the numbers of sites in two states are nearer, almost all the configurations are connected. Such a case is regarded as a liquid phase.

5. Concluding remarks

An attempt is made to analyze a pattern in this paper. The fields in science in which analyzing a pattern, shape or form is important have not sufficiently developed yet even now. Perhaps, it is the reason for it that we do not have enough concepts to do that. Science seems to have been leaving ill-quantifiable things untouched. Though much information should be contained in patterns, we are unable to draw it. It is desireble to develop such fields in cooperation with the scientists in various branches. Some difficulty is common to them.

[1] Y. Koshikiya and S. Hachisu: Lecture at COLLOID SYMPOSIUM OF JAPAN (Sept. 1982).
[2] G. H. Wannier: Phys. Rev. 79 (1950) 357.
[3] R. Kikuchi: Phys. Rev. 81 (1951) 988.
[4] C. Domb: Adv. in Phys. 9 (1960) 149.