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SPIN GLASS CHARACTER OF ANNEALED ISING SYSTEM ON TRIANGULAR LATTICE

Y. Kasai and A. Okiji

Department of Applied Physics, Osaka University, Suita, Osaka 565, Japan

The boundary concentration of a Mattis-type ordered phase (the hidden Mattis phase) on the antiferromagnet-rich side for the annealed system of the ±J model on the triangular lattice is estimated on a basis of the percolation problem of the Mattis order competing with the frustration. This is also considered an approximate boundary concentration of the spin glass phase for the quenched system.

1. Introduction

In previous papers [1,2], it was proposed that from the standpoint of Suzuki's real replica method [3] the annealed system in the case of the ±J model exhibits a Mattis-type ordered phase named the hidden Mattis phase which is ascribed to the same type of interaction as in the 1-state Potts model. It is expected that at the transition point of the hidden Mattis phase the dynamical critical phenomena such as the critical slowing down may appear although the free energy does not show any criticality [1]. This unfamiliar criticality is considered to stem from the property of the 1-state Potts model which is equivalent [4] to the percolation problem for the Mattis order (a type of Edwards-Anderson order [5]). It is interesting to point out that the spin glass of the quenched system has a similar property mentioned above. That is, there exists no order parameter [6,1] in the ordinary sense but remain some critical characters of the Mattis order in the decay property such as the slow relaxation of spin [7]. With the use of the critical values of the Ising model and the 1-state Potts model, we obtained the approximate phase diagrams (the concentration vs the temperature) with the hidden Mattis phase for some lattices [1]. However, for the triangular lattice which can not have the Néel state, the phase boundary of the hidden Mattis phase on the antiferromagnet-rich side was not determined definitely. In fact, the phase boundary [1] defined as the singular point of the residual entropy leads to an unreliable result that the hidden Mattis phase is broken by an infinitesimal frustration. The estimation of that boundary is interesting, because in this lattice structure the frustration which competes with the Mattis order may exist at the ground state despite being the annealed system. In order to obtain the more accurate value of the phase boundary, we propose the percolation problem defined on a frame of elementary-triangles which can express the Mattis order explicitly on the triangular lattice (see Fig.1).

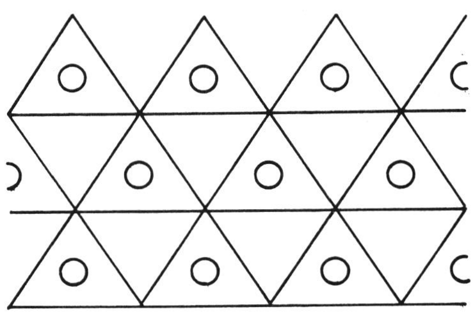

Fig. 1. The frame of elementary-triangles (indicated by the circles) on the triangular lattice.

2. The Annealed System

The grand partition function Ξ of the annealed system can be written as [1]

$$\Xi = \sum_{\{s_i=\pm 1\}} \prod_{<ij>} (x \exp[Ls_i s_j] + \exp[-Ls_i s_j])$$

$$= A^B \sum_{\{s_i\}} \prod_{<ij>} \exp[Ks_i s_j + K'(\Delta(s_i s_j)-1)]$$

$$= A^B \sum_{\{s_i\}} \prod_{<ij>} \exp[Ks_i s_j] = A^B Z(K) , \quad (1)$$

where

$$A^2 = (xe^L+e^{-L})(xe^{-L}+e^L) ,$$
$$e^{2K} = (xe^L+e^{-L})/(xe^{-L}+e^L) ,$$
$$e^{2K'} = (xe^L+e^{-L})(xe^{-L}+e^L)/(x+1)^2 , \quad (2)$$

$L=J/KT$, $\Delta(s_i s_j)$ is equal to 1 for $s_i s_j=\pm 1$ and 0 for a fictitious value $s_i s_j=0$, s_i denotes the Ising spin variable (± 1) on the i-th site, x the fugacity of +J bonds, B the total number of bonds and Z(K) the partition function of the Ising model with the interaction K. Though K' does not contribute to Z(K), it is used to determine the critical points of the hidden Mattis phase. The concentration p of the +J bonds can be expressed as

$$p = B^{-1} x\partial \ln\Xi/\partial x$$
$$= 2^{-1}(e^{2L}-e^{-2K})(e^{2L}-e^{-2L})^{-1}$$
$$\times[1+\varepsilon+(1-\varepsilon)e^{2K-2L}] , \quad (3)$$

where ε denotes the correlation function of the nearest neighbor spin pair and is defined as $(1/B)\ \ln Z(K)/K$. From (3), we can draw up the equi-K lines on the (p vs 1/L) plane in Fig.2. The critical concentration p_c is obtained as 5/6 by using the critical K and ε of the Ising model. The critical points B(1/L=1.009) and D(p=0.820, 1/L=1.202) in Fig.2 can also be obtained with the use of the critical parameters of the Ising model and the 1-state Potts model [1].

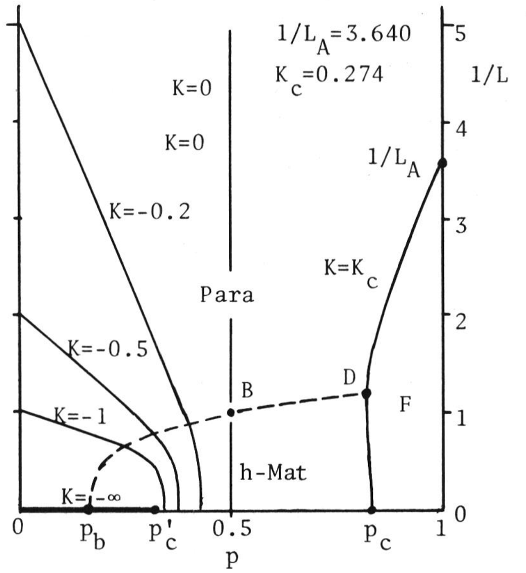

Fig. 2. The phase diagram, the concentration p of the +J bonds vs the temperature 1/L (=kT/J) with the equi-K lines, for the annealed system of the ±J model on the triangular lattice. The ferromagnetic, hidden Mattis and paramagnetic phases are denoted by F, h-Mat and Para, respectively. The p'_c denotes the singular point of the residual entropy (refer to Fig.2 and the paragraph 1).

3. The Residual Entropy

To investigate the ground state configuration of the annealed system with the frustration, we calculate the residual entropy. The entropy per bond, S, can be given as

$$S = K[B^{-1}\ell n\, Z(K) - K + p\{\frac{2L(1-\varepsilon)e^{2K-2L}}{1+\varepsilon+(1-\varepsilon)e^{2K-2L}} - \ell n\,\frac{2p}{1+\varepsilon+(1-\varepsilon)e^{2K-2L}}\}$$

$$+(1-p)\{\frac{(-2L)(1-\varepsilon)e^{2K+2L}}{1+\varepsilon+(1-\varepsilon)e^{2K+2L}} - \ell n\,\frac{2(1-p)}{1+\varepsilon+(1-\varepsilon)e^{2K+2L}}\}] \; . \qquad (4)$$

From the expressions (4) and (3), the following results for the residual entropy S_0 (S for $1/L \to 0$) can be obtained.

(a) $1 \geqq p > 1/3$ (K finite)

$$p = (1+\varepsilon)/2 \; . \quad (\varepsilon \text{ is a function of } K) \qquad (5)$$

$$S_0 = k[B^{-1}\ell n\, Z(K) - K\varepsilon] \; . \qquad (6)$$

(b) $1/3 \geqq p \geqq 0$ $(K \to -\infty,\ \varepsilon \to -1/3)$

$$e^{2K+2L} \to (1-3p)^{-1} \qquad (7)$$

$$S_0 = k[\delta + (1/3)\ell n(1/3) - p\,\ell n\, p - (1/3-p)\ell n(1/3-p)] \; , \qquad (8)$$

where $\delta = \lim_{K\to-\infty} [B^{-1}\ell n\, Z(K) - K] = 0.10768$.

In $1 \geqq p > 1/3$, the ground state is described by the frustrationless Mattis order. In $1/3 \geqq p \geqq 0$, the expression (8) is understandable from the following considerations. For p=0, an allowable spin configuration is such that one of the pair spins in each elementrary-triangle (see Fig.1) must be frustrated. The number $\exp[B\delta/k]$ is that of allowable spin configurations on the whole lattice at p=0. The frustrated bonds for each spin configuration stated above are replaced by the +J bonds as p is increased. The possible number of the replacements for each spin configuration at p is given by the combination number (B/3,Bp).*) Accordingly, the total number of possible configurations is given by the product of $\exp[B\delta/k]$ and (B/3,Bp). From this argument, the expression (8) can be derived again.

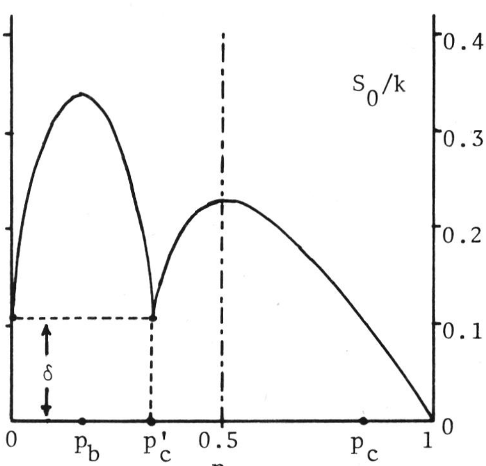

Fig. 3. The residual entropy S_0 per bond as a function of p for the annealed system of the ±J model on the triangular lattice.

4. The Percolation Problem

Furthermore, from the above discussion, the rate of frustrationless elementary-triangles at p $(1/3 \geqq p \geqq 0)$ can be given as

$$(B/3 - Bp)/(B/3) = 1-3p = p^* \; . \qquad (9)$$

In order to treat the annealed system as the percolation problem we assign the on-element to the frustrationless elementrary-triangle and the off-element to the frustrated one. The hidden Mattis phase may appear in the case where the on-elements percolate or the infinite cluster of on-elements is build up. Note that the on-off configuration does not appear completely at random. However we assume the complete randomness of the on-off configurations. Then, the percolation problem of the elementary-triangles is equivalent to the site percolation problem on the triangular lattice [8] for which the critical concentration is equal to 1/2. Therefore, from (9) we can consider $p^*=1/2$. That is, the boundary concentration p, say p_b, of the hidden Mattis phase on the antiferromagnet-rich side can be determined as 1/6.**)

5. Concluding Remarks

Using the boundary concentration p_b (=1/6) and other characteristic points obtained previously [1], we can draw up the schematic phase diagram Fig.2. This diagram can be regarded approximately as that of the quenched system as mentioned in the first paragraph (see ref.[1]). The spin glass of this type disappears in the frustration-rich region. This is quite different from the results proposed by Aharony and Binder [9].

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*) The symbol (n,m) denotes the combination number usually expressed by $\binom{n}{m}$.

**) The boundary value p_b depends on the lattice structure, but it is independent of n for the n-replicated system [2] and also of α for the $(J,-\alpha J)$ model.