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VECTOR SPIN GLASS IN A MAGNETIC FIELD

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The effect of a field, applied or spontaneous, on the ordering and response of an isotropic classical vector spin glass is considered within replica mean field theory, using Parisi treatment of replica symmetry breaking. For small mean exchange there is a single phase transition marking the onset of transverse spin-glass order, which is accompanied by longitudinal and transverse magnetic irreversibility. At a lower temperature there is a crossover in the longitudinal irreversibility. For large enough mean exchange spontaneous ferromagnetism occurs. There are two ferromagnetic phases, one collinear and reversible, the other non-collinear and irreversible.

The fundamental ingredients for spin-glass behaviour [1] have been recognized [2] as the combination of frustration [3] and quenched spatial randomness. These features can be present already in an Ising model [4] and in view of the greater mathematical simplicity of Ising systems most work to date has concentrated on them. Vector models have received less attention since they were expected to show qualitatively similar behaviour to the Ising models, the spins simply being frozen throughout a larger space. On a gross conceptual level this is true. It is also true microscopically for an isotropic system. However, as first pointed out by Gabay and Toulouse [5], the addition to an otherwise isotropic system of a magnetic field, applied or spontaneous, gives rise to qualitatively new features as a consequence of the resultant anisotropy between the field direction and directions orthogonal to it. This paper is concerned with those features.

Although a precise understanding of spin-glass experiments, real or Monte Carlo, is still lacking, two important areas of comprehension/correlation can be identified, one a qualitative appreciation of the state structure (entropically extensive multiplicity of metastable states with a quasi-continuous range of intervening potential barriers), the other a folklore mapping between the predictions of replica mean field theory (with replica symmetry (RS) breaking) and experimental consequences such as field-cooled or zero-field-cooled responses. In the present paper only the replica mean field theory will be considered, together with its predictions for experiment.

We base our analysis on the m-vector Sherrington-Kirkpatrick (SK) [4] model in a field

$$\mathcal{H} = -\sum_{(ij)} J_{ij}\, \underline{S}_i \cdot \underline{S}_j - \sum_i \underline{H} \cdot \underline{S}_i \qquad (1)$$

where the J_{ij} are quenched, independently random exchanges distributed with mean J_o/N, variance $J/\sqrt{N}$. We choose units with $J=k_B=1$, $S^2=m$. Mean field theory [4] is believed to be exact for this model and its solution yields the mean-field solution to a short-range model. We present here only qualitative results, which are based on an extension [5,6] of the now-conventional replica analysis [2, 4], including replica symmetry breaking within a Parisi-like scheme [7-10]; for details see references.

In replica space the order parameters are

$$q_\lambda^{\alpha\beta} = \lim_{n\to 0} \langle S_\lambda^\alpha S_\lambda^\beta \rangle_n \, , \qquad m_\lambda^\alpha = \lim_{n\to 0} \langle S^\alpha \rangle_n \tag{2}$$

where the λ label Cartesian coordinates and the $\langle\ \rangle_n$ designates a thermal average within an n-replicated effective system; $\alpha,\beta = 1,..n$. The relevant spin-glass order parameters are the $q_\lambda^{\alpha\beta}$ with $\alpha \neq \beta$; the $q_\lambda^{\alpha\alpha}$ are secondary quadrupolar parameters which have no singular behaviour. Within RS approximation all pairs of different α,β are equivalent and $q_\lambda^{\alpha\beta} = \overline{\langle S_\lambda \rangle^2}$, where $\langle\rangle$ refers to a thermal average of the real system and the bar a spatial average. At high temperature this approximation is exact, but it breaks down at a critical temperature identified as the spin-glass transition. Breakdown signals history-dependence, physically due to the metastable state and barrier structure.

The Parisi scheme replaces the $q_\lambda^{\alpha\beta} (\alpha \neq \beta)$ by functions $q_\lambda(r)$; $0 < r < 1$; Sompolinsky [11] identifies the r-range $(1 - 0)$ as relating to a physical measurement time-scale $(0 - \infty)$ with $r = 1$ equivalent to $t = 0$. Within this scheme replica-symmetry corresponds to a flat $q_\lambda(r)$. In general, two local susceptibility expressions can be studied, one calculated directly from the magnetization in an applied field, to be referred to as the equilibrium susceptibility, and given by

$$\chi_\lambda^E = T^{-1}\left(q_\lambda^{\alpha\alpha} - \int_0^1 q_\lambda(r) \right) \, ; \tag{3}$$

the other evaluated from fluctuations of the unperturbed system, referred to as the linear response susceptibility and given by

$$\chi_\lambda^{LR} = T^{-1}\left(q_\lambda^{\alpha\alpha} - q_\lambda(max) \right) = T^{-1}\left(q_\lambda^{\alpha\alpha} - q_\lambda(1) \right). \tag{4}$$

χ^E, χ^{LR} are identified with the experimental field-cooled (FC) and quasi-instantaneous zero-field-cooled (ZFC) local susceptibilities. Their difference is known as the irreversible susceptibility, χ^I. It is clearly non-zero whenever $q(r)$ is not flat, i.e. when replica symmetry is broken and the multiplicity of the metastable states becomes relevant.

There are only two independent $q_\lambda(r)$, one in the field direction, which we denote $q_L(r)$, one transverse, which we denote $q_T(r)$.

Let us consider first $J_o = 0$. Above a critical temperature T_1 [5] $q_T(r)$ is zero and $q_L(r)$ is flat (but non-zero if H is non-zero). At T_1 transverse freezing occurs and $q_T(r)$ becomes non-zero continuously. Both $q_T(r)$ and $q_L(r)$ are non-flat beneath T_1 so that both transverse and longitudinal local susceptibilities exhibit irreversibility. A full expression for T_1 will be found in Ref. 6;

to lowest order in H it is given by [12]

$$T_1 = 1 - (m^2 + 4m + 2) H^2/4(m+2)^2 + \cdots \qquad (5)$$

In their original paper Gabay and Toulouse [5] incorrectly found no replica symmetry breaking until a lower temperature $T_2 = 1 - \alpha H^{2/3}$, but this was because the full stability matrix was not considered. In fact, we now predict a crossover in the longitudinal irreversibility at a relative temperature $\tau = (1-T) \sim H^{2/3}$. In the Ising limit (m=1) this becomes the Almeida-Thouless [13] transition.

Beneath T_1, $q_T(r)$ and $q_L(r)$ satisfy complicated coupled integro-differential equations [8,9] whose solution for general T is highly non-trivial. The nature of the irreversibility and the crossover can, however, be described qualitatively in a simple manner. We restrict our comments to small $\tau = (1-T)$ and H. In general, for $T < T_1$, $q_{L,T}(r)$ are constant above some $r^* \sim \tau$ and take the values

$$q_T(1) \sim \tau \ , \quad q_L(1) \sim \tfrac{1}{4}\left[((1-T^2)^2 + 8H^2)^{1/2} + (1-T^2) \right], \qquad (6)$$

while $q_T(0)$ is zero. It immediately follows that $\chi_T^{\,I} \sim \tau^2$, essentially like a rescaled (m-1)-dimensional isotropic model without a field [14]. The small $q_L(r)$ behaviour is, however, quite different depending upon whether τ is large or small compared with a value τ^* scaling as $H^{2/3}$. This is because stability requirements impose a maximum effective-pinning value on $q_L(0)$ proportional to $H^{2/3}$; we call it q_L^*. For $\tau \ll \tau^*$

$$q_L(0) \ll q_L^* \ , \quad q_L(r^*) - q_L(0) \sim \tau^2 (q_L(1)^2/H^2) \qquad (7)$$

so that $\chi_L^I \sim \tau^3 (q_L(1)/H)^2$, much weaker than χ_T^I, or than χ_L^I for a system with no field. On the other hand, for $\tau > \tau^*$ $q_L(0)$ is pinned at q_L^* and χ_L^I becomes stronger, passing over for $1 \gg \tau \gg \tau^*$ to $O(\tau^2)$ behaviour (less corrections of $O(H^{4/3}, \tau H^{2/3})$). It is, perhaps, worth noting the corresponding situation for an Ising system; again there is a pinning $q^* \sim H^{2/3}$ but, since there is no higher transition, q(r) is flat below the corresponding τ^* which therefore signals a phase transition (onset of irreversibility) rather than crossover.

The predicted magnetization behaviour is illustrated schematically in Fig. 1 [10]. Solid curve A represents the equilibrium (or field-cooled) magnetization. Curve B qualitatively indicates the difference expected for zero-field cooling. As for an Ising model in a field deviations from a Curie law occur already above the transition temperature with, for $T > T_1$,

$$M = (H/T)\left\{ 1 - \tfrac{1}{4}\left(((1-T^2)^2 + 8H^2)^{1/2} + (1-T^2) \right) + \cdots \right\}. \qquad (8)$$

Also like in an Ising model, one finds a low temperature FC magnetization with the temperature-independent form

$$M = H\left\{ 1 - \alpha H^{4/3} + \cdots \right\} \qquad (9)$$

and irreversibility

$$M(FC) - M(ZFC) \sim (\tau - \tau^*) \quad \text{for } \tau > \tau^* \sim H^{2/3} . \qquad (10)$$

As far as a longitudinal probe is concerned the effects of the phase transition at T_1 are relatively weak until a temperature more like T_2 of Gabay and Toulouse, but the irreversibility may be observable with care in a sensitive experiment.

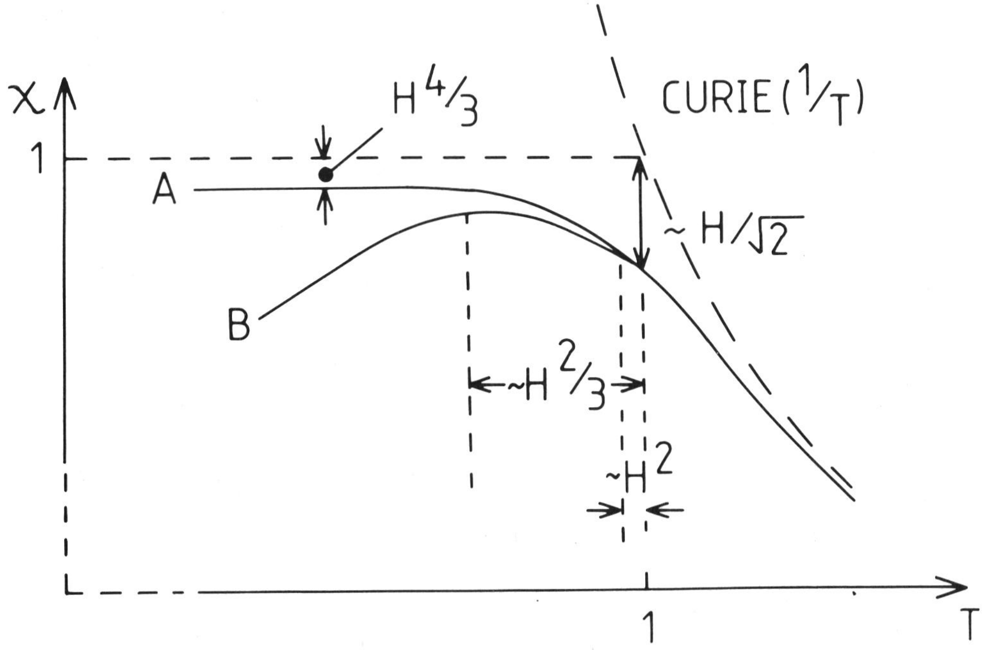

Figure 1 Schematic magnetization of a vector spin glass in a field H. Curve A represents field cooled, curve B zero-field cooled magnetization.

Since the predictions of references 13 and 5 there have been several experiments studying the field-cooled magnetization [15] and the irreversibility [16-19] in a field; systems have included crystalline Ag Mn [15,16], $Eu_{.4}Sr_{.6}S$ [18] and $Cd_{.45}Mn_{.55}Te$[19], and amorphous $(Fe_{.64}Mn_{.36})P_{16}B_6Al_3$[17]. Outside the region of weak longitudinal irreversibility, and for not too small fields, they have exhibited behaviour qualitatively as in equations (8)-(10), although a scaling factor relating experimental to theoretical field strength is needed to make numerical fits; experimental field sensitivity is of order 10-20 times greater than that predicted by mean field theory. However, only one of these experiments [19] appears to show both the characteristic temperatures; i.e. the onset of weak as well as strong longitudinal irreversibility. The non-observation of a weak region in refs. 16-18 may be a question of experimental sensitivity/accuracy or may be due to the effects of random anisotropy which can suppress the q_T-ordering transition leaving only the replica-symmetry breaking transition, which will be Ising-like for fields small compared with the anisotropy [17,20]. For very small fields all the experimental magnetizations exhibit a peak around T=1, which is not predicted by present theory.

When J_o is large enough, spontaneous ferromagnetism is possible. Toulouse mapping [21] shows that two ferromagnetic phases are to be expected for $J_o>1$; a higher temperature collinear reversible state and a lower temperature canted irreversible state, with the latter having an upper region of weak longitudinal irreversibility and a lower one of strong longitudinal irreversibility. The phase diagram is like that of Gabay and Toulouse but with a single "mixed" phase and only a crossover separating the regions they labelled M_1, M_2. Experimental evidence for two types of ferromagnetism, collinear and non-collinear, separated by a phase transition is to be found in resistance and magnetoresistance [22] and in Mössbauer experiments [23].

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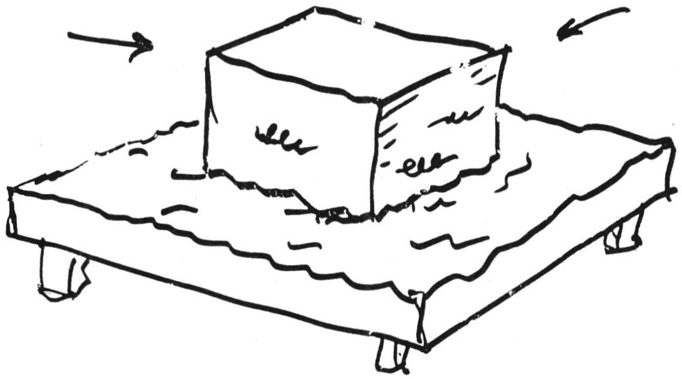
FROM THE SUMMARY TALK (3)

Spin Glass Pictures



Tohfu (bean-curd, a typical Japanese food, soft and white coloured):
A model of spin glass?