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ANDERSON LOCALIZATION OF TWO-DIMENSIONAL SYSTEM UNDER STRONG MAGNETIC FIELDS

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The dynamical diffusion coefficient of two-dimensional electron-impurity system under quantizing magnetic fields is determined through a diagramatic self-consistent calculation of the density correlation function. Extended states do exist but only at one energy for each Landau subband, at which the real part of the retarded single-particle Green function vanishes.

1. Introduction

The Anderson localization may be regarded as a phase transition where the Fermi energy plays the role of the temperature. If we consider an analogy with spin systems, the insulating state corresponds to the paramagnetic phase, the localization length to the correlation length, and the metallic state to some ordered phase. There are several suggestions about the equivalence between the Anderson localization problem and a certain class of spin systems [1,2]. In this paper we discuss a very interesting behaviour of the dynamical diffusion coefficient of two-dimensional electron system moving in random static potentials under strong magnetic fields. If we can construct any spin system equivalent to the present electron system, a very peculiar new type of phase transition will be expected in the spin system. We do not go into the equivalence problem any more in the present paper.

Recent theoretical investigations on the Anderson localization in two-dimensional systems [3-7] derived a conclusion that particles scattered by random potentials are always localized irrespective of the strength of the potential fluctuation as far as the system has the time reversal sysmmetry. The magnetic field violates the time reversal symmetry and has been shown to weaken the localization [8,9]. We have also shown that the weak magnetic fields will not give rise to extended states in the two-dimensional system [10]. Thus there occurs a naive question whether strong magnetic fields break the two-dimensional localization or not. This problem is particularly important recently, since the quantized Hall effect in MOSFET has been pointed out to be used in accurate determination of the fine structure constant [11,12]. In order to have quantized Hall conductivity, there must exist at least one extended state [13].

2. Self-Consistent Determination of Diffusion Coefficient

In order to derive a self-consistent equation for the dynamical diffusion coefficient, we consider the q- and ω-dependent density respense function $\chi(q,\omega)$, which can be written generally in the following form [6],

$$\chi(q,\omega) = \phi(q,\omega,E) + N(E) + O(\omega,q^2) \quad , \tag{2.1}$$

where ϕ is the density relaxation function, $N(E)$ the density of states at the Fermi energy E. We restrict ourselves to the absolute zero of temperature and discuss the Fermi energy dependence of physical quantities. Because of the particle number conservation ϕ must have the following form in the small q limit,

$$\phi(q,\omega,E) = -N(E)/[\omega + iD(\omega,E)q^2] \ , \tag{2.2}$$

where D has the meaning of the diffusion coefficient. Once we know the density response function $\chi(q,\omega)$, it is straightforward to obtain the dynamical conductivity $\sigma_{xx}(\omega,E)$;

$$\sigma_{xx}(\omega,E) = e^2 \lim_{q\to 0}(-i\omega/q^2)\chi(q,\omega) = e^2 N(E) D(\omega,E) \ . \tag{2.3}$$

The last equality in nothing but the Einstein relation.

In the following we take a model where free electrons are moving in a two-dimensional space with randomly distributed short-range potentials (strength u, concentration n_i) under a strong magnetic field perpendicular to the system. To simplify the problem, we consider the case where the Landau subband width Γ is much smaller than the cyclotron frequency ω_c and where the Fermi energy lies within the lowest Landau subband. The details of the derivation of the self-consistent equation will be published else where [14,15]. Here we explain the essence of the procedure. In the present situation, the density relaxation function is expressed in terms of the impurity average of the product of the retarded and advanced simgle-particle Green functions which are generally non-diagonal in the Landau state representation. Through the standard diagramatic method, this average of the product of Green functions can be decomposed into a sum of products of averaged single-particle Green functions which are now diagonal in the Landau representation and, within the present model, independend of the center coordinate of the cyclotron motion. Thereby the concept of the irreducible vertex correction is naturally introduced and the diffusion coefficient can be expressed in terms of the irreducible vertex correxion. The exact treatment of the irreducible vertex correction is usually impossible. In order to derive a closed self-consistent equation for $D(\omega,E)$, we calculate the irreducible vertex correction in an approximation which takes account of the electron motion in an effective diffusive medium characterized by the density relaxation function of the form of eq. (2.2). In this way a self-consistent equation for the dynamical diffusion coefficient is obtained. The equation itself is too lengthy to be written down here and is omitted [14]. In the following section we discuss about the solution of the equation.

3. Dynamical Diffusion Coefficient and Localization Length

The most prominent property of the self-consistent equation discussed above is that the logarithmically singular term due to the diffusion pole of ϕ does not vanish as far as $S(E) \equiv G^R(E) + G^A(E) \neq 0$; here G^R and G^A are the retarded and advanced Green functions of the lowest Landau subband. Because of this singurality, the losution $D(\omega,E)$ of the self-consistent equation with $S(E) \neq 0$ has to be zero at $\omega = 0$. In the small ω region, the solution has the following form,

$$D(\omega,E) = D_0(E)\{-i\Omega A_1(E) + \Omega^2 A_2(E) + O(\Omega^3)\} \ , \tag{3.1}$$

where $D_0(E) = 2\pi\ell^2 n_i u^2 N(E)$, $\Omega = \omega\ell^2/D_0(E)$ with $\ell^2 = c\hbar/eH$ (ℓ the magnetic length), and A_1 and A_2 are positive functions and their values are calculated numerically in general. Especially when E approaches E_0 at which $S = 0$, A_1 behaves as

$$A_1(E) = 2^{-1}\exp[\gamma + 1 + P(E)/(S(E))^2] \ , \tag{3.2}$$

with $P(E) \equiv G^R(E)G^A(E)$ and γ Euler's constant, and A_2 is roughly proportional to A_1^2. The quantity $P(E)/(S(E))^2$ may be approximated by $\alpha\Gamma^2/(E-E_0)^2$ with α a constant of order unity.

At $E = E_0$, the self-consistent equation has a finite solution even for $\omega = 0$, and the behaviour of $D(\omega,E_0)$ in the small ω region is found to be

$$D(\omega,E_0) = D_0(E_0)\{a - R_1|\Omega| - iI_1\Omega[\ln|\Omega| + b] + o(\Omega)\} \ , \tag{3.2}$$

where Ω should be evaluated at $E = E_0$, and a, b, R_1, and I_1 are positive constants of order unity. In figs. 1 and 2, the numerical solutions of the self-consistent equation are depicted as the function of the frequency with the Fermi

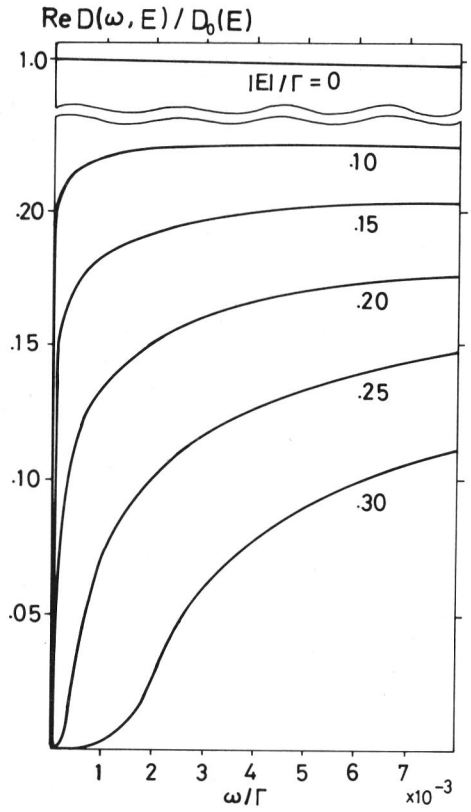

Fig. 1: The real part of the dynamical diffusion coefficient.

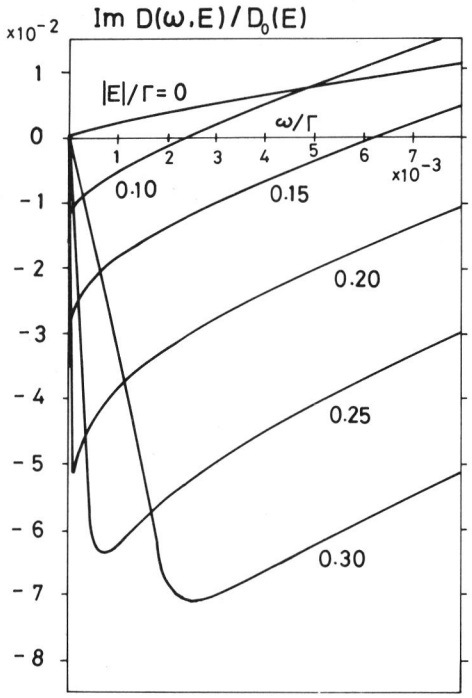

Fig. 2: The imaginary part of the dynamical diffusion coefficient.

energy as a parameter (E is measured from E_0). In the calculation we have used the single-particle Green functions obtained by the self-consistent Born approximation for simplicity. Even if better approximations were used for the single-particle Green functions, there would be no qualitative change in the frequency and energy dependences of $D(\omega,E)$. From Figs. 1 and 2 we find that, reflecting the singular ω dependence at $E = 0$, both of Re D and Im D show very rapid change in the small ω region, especially when E tends to zero.

The result described above indicates that the extended states exist only at $E = E_0$ where the real part of the retarded Green function chages the sign, and that all other states with $E \neq E_0$ are localized. From a very general discussion A_1 is found to be equal to the square of the localization length ξ if $D(\omega,E)$ has the form of eq. (3.1). Thus we find that the energy dependence of ξ is very singular when E goes to E_0. In Fig. 3, the result of the numerical calculation for ξ is depicted. Note that ξ^{-1} is very snall at small values of E (E is again measured from E_0) but that it never becomes zero as far as $E \neq 0$.

5. Concluding Remark

By a self-consistent treatment of the dynamical diffusion coefficient, we have shown that all the states in the lowest Landau subband except for those with energy E_0 at which the real part of the retarded Green function vanishes are localized in the sense that the static diffusion coefficent is zero. When E approaches E_0, the localization length behaves as $\ell\exp[c\Gamma^2/(E-E_0)^2]$. This

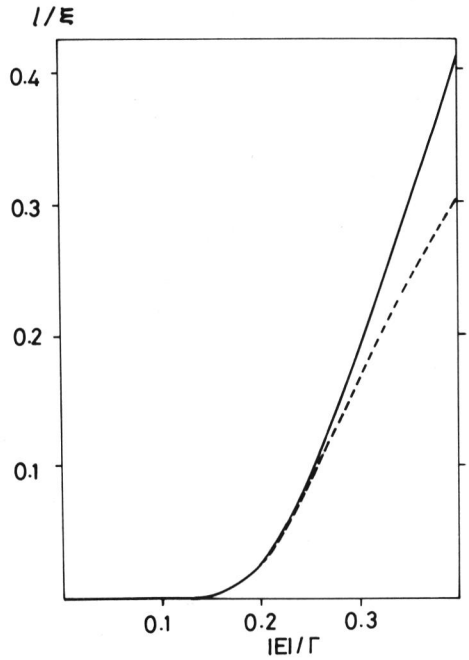

Fig. 3: The energy dependence of the inverse localization length. The broken line is the extrapolation of the analytic expression appropriate for $|E|/\Gamma \ll 1$ (see eq.(3.2)).

singular energy dependence of the localization length is reflected in the very rapid change of $D(\omega,E)$ in the small ω region when $E \sim E_0$. The present calculation can be extended to the case where the Fermi energy lies in a higher Landau subband, and we have obtained similar conclusion that only the states at the subband center are extended. Thus, as changing the Fermi energy, we can expect a very peculiar behaviour of the localization length, i.e., it diverges whenever the Fermi energy passes through the center of a subband. The extended states do exist but the measure of those states in the energy space is vanishingly small. The present result is consistent with the observation of the quantized Hall effect in two-dimensional electron system under strong magnetic fields.

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