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Relationships between Tensor Analyzing Powers in Elastic Scattering of Spin 1 and 3/2 Particles†

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Relationships between second-rank tensor analyzing powers are derived by the invariant-amplitude method for elastic scattering of spin 1 and 3/2 particles. They explain a characteristic feature of A_{yy}, A_{xx} and A_{xz} measured in the scattering of deuterons. Analyzing powers of ^{6}Li scattering by four-channel calculations are found to satisfy the relation. The relationship formulae are applied to experimental data on T_{20}, T_{21} and T_{22} in scattering of ^{7}Li by ^{58}Ni with success. The theory is extended to third-rank tensor analyzing powers.

§1. Introduction

In nuclear polarization phenomena, effects of individual spin-dependent interactions are sometimes obscure due to complications by mixed effects of several kinds of spin-dependent interactions: for example, tensor analyzing powers in the scattering of deuterons are produced by tensor interactions, higher orders of spin-orbit ones and their admixtures. As was shown in ref.1, the invariant-amplitude(IA) method gives a guide-post for solving the complications, where scattering amplitudes are decomposed according to the tensor rank in the spin space. Although each amplitude with a definite tensor rank includes still higher-order effects of various spin-dependent interactions, it is easier to investigate contributions of a particular spin-dependent interaction in this amplitude than in the original one, because of the specification of the tensor rank. These features resemble those in the theoretical works in ref.2.

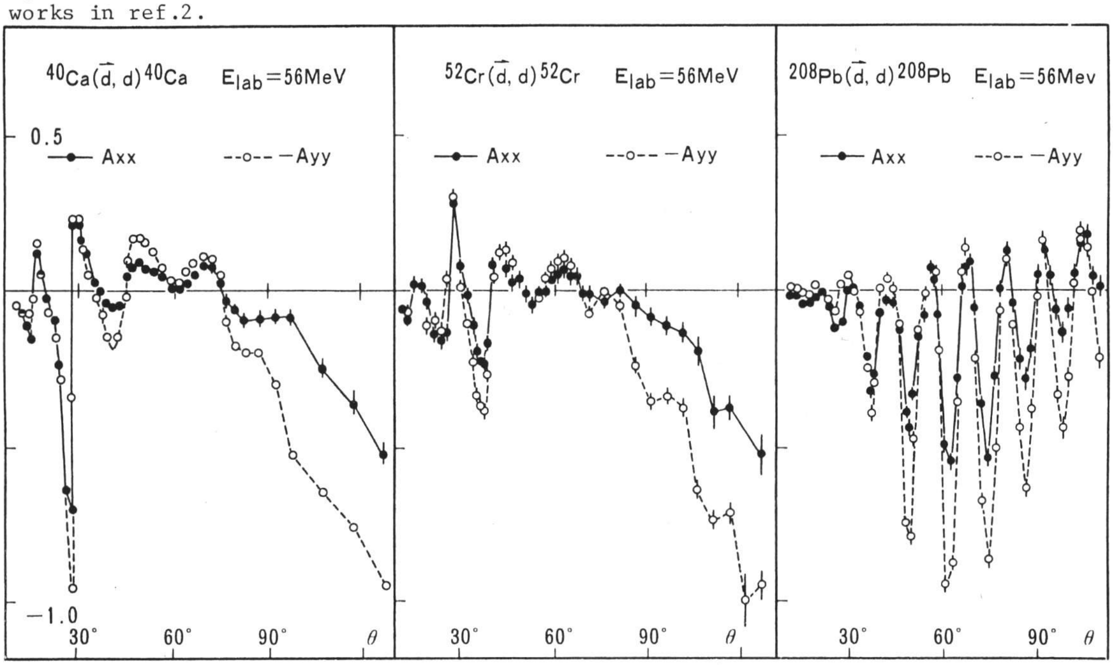

Fig.1. Comparison between A_{xx} and $-A_{yy}$ measured for ^{40}Ca, ^{52}Cr and ^{208}Pb targets.

† Presented by M. Tanifuji.

This note studies the tensor analyzing powers A_{xx}, A_{yy} and A_{xz} for deuteron elastic scattering by the IA method and discuss higher-order effects of spin-dependent interactions. The analyzing powers A_{xx} and A_{yy} measured at E_d=56 MeV[3] have similar angular distributions for each target, which are characterized in a middle angular region as

$$A_{yy} \simeq -2A_{xx} , \tag{1}$$

examples of which are shown in Fig.1. The theory derives this relation and a more refined one in a way free from theoretical parameters. The latter relation is transformed into the spherical representation, which is shown to describe the observed T_{20}, T_{21} and T_{22} in ^{58}Ni($^7\vec{\text{Li}}$, ^{7}Li)^{58}Ni very well. ^{58}Ni($^6\vec{\text{Li}}$, ^{6}Li)^{58}Ni and ^{12}C($^7\vec{\text{Li}}$, ^{7}Li)^{12}C are also investigated. Finally, extensions to third-rank tensor analyzing powers are discussed for ^{120}Sn($^7\vec{\text{Li}}$, ^{7}Li)^{120}Sn.

§2. Tensor Analyzing Powers by Invariant Amplitudes

The general formula for transition amplitudes[1] is applied to elastic scattering of a spin 1 particle by a 0^+ target. Projectiles of spin 3/2 are treated in a similar way. The transition amplitude $\langle \nu_f, \vec{k}_f | M | \nu_i, \vec{k}_i \rangle$, where $\vec{k}_i(\vec{k}_f)$ is the momentum and $\nu_i(\nu_f)$ the z-component of the spin in the initial(final) state, is given by the invariant amplitude F_{Kr},

$$\langle \nu_f, \vec{k}_f | M | \nu_i, \vec{k}_i \rangle = \sum_K (-)^{1-\nu_f} (11\nu_i\ -\nu_f | K\kappa) \sum_{r=\bar{K}-K}^{K} [C_r(\Omega_i) \times C_{\bar{K}-r}(\Omega_f)]^K_\kappa\, F_{Kr}, \tag{2}$$

where $\bar{K}$ =K for K=even and $\bar{K}$=K+1 for K=odd and $\Omega_i(\Omega_f)$ is the solid angle of $\vec{k}_i(\vec{k}_f)$. Let us define the scalar amplitude U, the vector one S and the tensor ones T and T',

$$U=\sqrt{3}\,F_{00},\quad S=F_{11}\sin\theta/\sqrt{2}, \tag{3}$$
$$T=(\sqrt{6}\,F_{20}\cos\theta + F_{21})\sin\theta/\sqrt{2},\quad T'=\sqrt{3/8}\,F_{20}\sin^2\theta,$$

where θ is the angle between $\vec{k}_i$ and $\vec{k}_f$, the coordinate axes being chosen as $z // \vec{k}_i$ and $y // \vec{k}_i \times \vec{k}_f$. The amplitudes U, S and T(T') are related to the central, spin-orbit and tensor interactions, respectively, in their first order and thus U is dominant and others are relatively small. Polarization observables are described by these amplitudes. Neglecting U-independent terms,

$$A_{xx}=2\sqrt{2}Re(\sqrt{2}UT'^* - UT^*\cot\theta)/9\sigma,\quad A_{yy}=2\sqrt{2}Re(-2\sqrt{2}UT'^* - UT^*\cot\theta)/9\sigma, \tag{4}$$
$$A_{xz}=\sqrt{2}Re(UT^*)/3\sigma,$$

where σ is the cross section. Using these equations, we obtain

$$A_{yy}\sin\theta = -2A_{xx}\sin\theta - 2A_{xz}\cos\theta, \tag{5}$$

which is transformed into the spherical representation,

$$\sqrt{3/2}\,T_{20}\sin\theta = T_{22}\sin\theta - 2T_{21}\cos\theta. \tag{6}$$

These formulae can be derived for spin 3/2 particles in the same approximation. Exact relationships which correspond to (6) are rather complicated and will be discussed in ref.4. In the following, we will investigate the validity of the approximate formulae derived above in scattering of deuterons, ^{6}Li and ^{7}Li and emphasize that they provide a simple method to obtain information about spin-dependent interactions directly from experimental data.

§3. Elastic Scattering of Deuterons and ^{6}Li

As was pointed out in sec.1, the relationship between the measured A_{xx} and A_{yy} is described by eq.(1). This relation is derived from eq.(5) by neglecting $2A_{xz}\cos\theta$. The justification of this approximation may be provided by $\cos\theta \simeq 0$ in the neighborhood of $\theta \simeq 90°$, being helped by the smallness of A_{xz}. Without this approximation, one can examine directly the validity of eq.(5), by comparing the left-hand side(LHS) and the

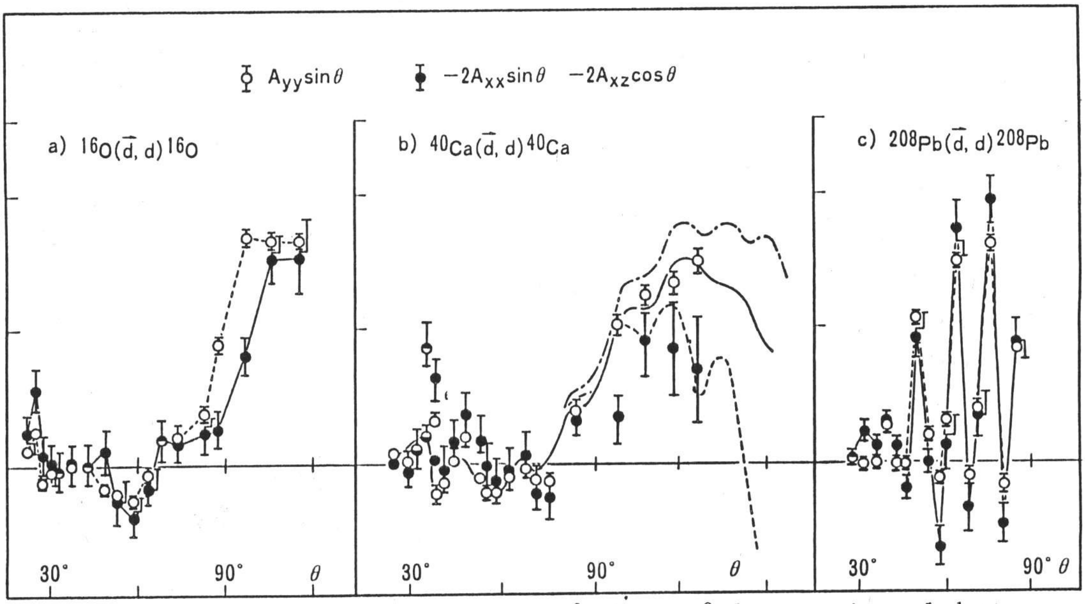

Fig.2. Comparison between $A_{yy}\sin\theta$ and $-2A_{xx}\sin\theta$ $-2A_{xz}\cos\theta$ for experimental data. In a) and c), the solid and dashed lines are for guiding eyes. In b), the lines are the optical-model calculations[3]. The solid line describes $A_{yy}\sin\theta$ calculated with a tensor potential and the dashed(with tensor) and the dash-dotted(without tensor) lines are for $-2A_{xx}\sin\theta$ $-2A_{xz}\cos\theta$.

right-hand side(RHS) on inserting the experimental data. The data[3] approximately satisfy eq.(5) in heavy targets but not at large angles in light targets, examples of which are shown in Fig.2. This means that effects of higher orders of spin-dependent interactions are almost negligibly small in heavy targets but are important in light targets. In exact calculations, A_{xx} and Axz include $Re(S^*T)$ but Ayy does not. The difference between the LHS and the RHS in eq.(5) is just $3Re(S^*T)/\sigma\sin\theta$, which will strongly be affected by the presence of tensor interactions. Fig.2 contains two kinds of optical-model calculations for the Ca target; (A) one including central and spin-orbit potentials, and (B) the other including central, spin-orbit and tensor ones[3]. They give almost the same values for the LHS, which is shown by the solid line for (B), but quite different ones for the RHS at large angles, where the dash-dotted line and the dashed line are for cases (A) and (B), respectively. The latter calculation reproduces the data dramatically, showing the importance of the tensor potential.

Since there are no available data for ^{6}Li scattering, we will study the validity of eq.(6) for ^{6}Li by inserting calculated T_{20}, T_{21} and T_{22} for ^{58}Ni targets at E_{lab}=20 MeV. The calculations are performed by the coupled-channel method with the cluster-folding model for ^{6}Li-nucleus interactions[5] where ^{6}Li virtual excitations to the 3^+,

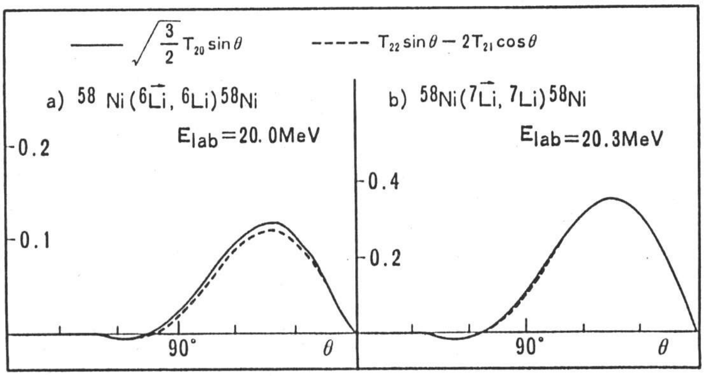

Fig.3. Comparison between $\sqrt{3/2}T_{20}\sin\theta$ and $T_{22}\sin\theta$ $-2T_{21}\cos\theta$ in four-channel calculations(see text). Fig.a) describes ^{58}Ni($^6\vec{\text{Li}}$, ^{6}Li)^{58}Ni and Fig.b) describes ^{58}Ni($^7\vec{\text{Li}}$, ^{7}Li)^{58}Ni.

2^+ and 1^+ excited states are taken into account. The interaction parameters are the same as those in ref.5, by which the cross-section and vector-analyzing-power data are well reproduced. The comparison between the LHS and the RHS of eq.(6) thus obtained is shown in Fig.3a. The agreement between the both sides is quite good.

§4. Elastic Scattering of ^{7}Li and Third-Rank Tensor Analyzing Powers

In the $^7Li+^{58}Ni$ scattering at low energies, the experimental data of tensor analyzing powers[6] can be described by the following formulae in good approximation [5,6], which is shown in Fig.4, for example at E_{lab}=20 MeV.

$$T_{20}=(1-3\sin^2\frac{\theta}{2})\,{}^{T}T_{20},\quad T_{21}=-\sqrt{\frac{3}{2}}\sin\theta\,{}^{T}T_{20},\qquad (7)$$
$$T_{22}=-\sqrt{\frac{3}{2}}\cos^2\frac{\theta}{2}\,{}^{T}T_{20},\quad \text{where}\quad {}^{T}T_{20}=-(T_{20}+\sqrt{6}T_{22})/2.$$

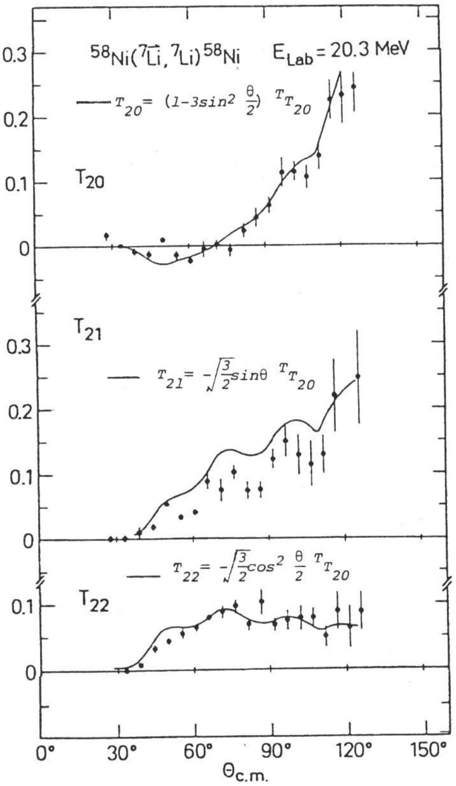

Fig.4. Comparison between the experimental data[6] and calculations by eq.(7) in $^{58}Ni(^7\vec{Li},{}^7Li)^{58}Ni$. For ${}^{T}T_{20}$, the experimental data are used.

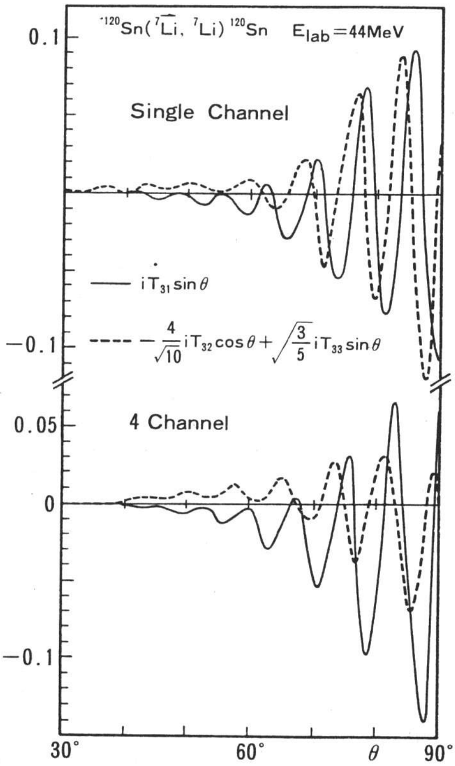

Fig.5. Comparison between $iT_{31}\sin\theta$ and $-\sqrt{8/5}iT_{32}\cos\theta+\sqrt{3/5}iT_{33}\sin\theta$ in $^{120}Sn(^7\vec{Li},{}^7Li)^{120}Sn$. The lines are obtained by the calculations (see text).

As is easily seen, eqs.(7) satisfy eq.(6) exactly. Thus the higher-order effects of spin-dependent interactions are quite small in this case. In fact, the coupled-channel calculations[5] with the cluster-folding model for ^{7}Li, which take account of the ground state and the $1/2^-$, $7/2^-$ and $5/2^-$ excited states for ^{7}Li and fit the data nicely, make only a very small difference between the LHS and the RHS in eq.(6), which is shown in Fig.3b.

The scattering of ^{7}Li by ^{12}C at E_{lab}=20 MeV is investigated in a similar way. The agreement between the data on T_{20}, T_{21} and T_{22} and eqs.(7) is not as good as with the ^{58}Ni target, at large angles[8]. Accordingly, eq.(6) loses its validity for $\theta \gtrsim 80°$. These disagreements may arise from strong couplings to the excitation of the target nucleus, which will become effective due to the low coulomb barrier of ^{12}C[8]. The details of these effects will be discussed in ref.7.

Third-rank tensor analyzing powers of ^{7}Li are very interesting, because they appear only for spin$\geq$3/2 projectiles. In a way similar to that in §2, the following relationship is derived by neglecting U-independent terms in the analyzing powers,

$$iT_{31}\sin\theta = -\sqrt{\frac{8}{5}}\,iT_{32}\cos\theta + \sqrt{\frac{3}{5}}\,iT_{33}\sin\theta. \qquad (8)$$

Since the experimental data exist only for ${}^{T}T_{30}=-(\sqrt{3}iT_{31}+\sqrt{5}iT_{33})/2$ for ^{120}Sn targets, we will examine the above relation by inserting the calculated analyzing powers for both sides. The calculation is performed in the coupled-channel frame by using the double-folding interactions[8] and it fits the data on σ, iT_{11}, T_{20}, T_{21}, ${}^{T}T_{20}$ and ${}^{T}T_{30}$ for both elastic and inelastic scattering simultaneously[9]. The comparison between the LHS and the RHS of eq.(8) is shown in Fig.5, where in single-channel calculations both sides of eq.(8) are almost similar to each other though small shifts of shapes are observed. In the four-channel calculations, virtual excitations of ^{7}Li destroy the similarity between the LHS and the RHS. By the detailed study of the calculation, it becomes clear that the tensor amplitudes are enhanced by the virtual excitations and sometimes compete with the scalar amplitude U. Thus, the higher-order terms of the spin-dependent interactions are not negligible .

§5. Summary

The invariant-amplitude method derives a relationship between the tensor analyzing powers A_{xx}, A_{yy} and A_{xz} or the one between T_{20}, T_{21} and T_{22} under the assumption of weak spin-dependent interactions. These relationships explain the feature of the experimental data of deuteron elastic scattering at 56 MeV. The deviation of the data from the above relation is understood as effects of tensor interactions. It is shown that the relationship also holds for the observed tensor analyzing powers in elastic scattering of ^{7}Li by ^{58}Ni at low energies. The coupled-channel calculation including projectile virtual excitations proves the relationship to be valid in ^{6}Li elastic scattering by ^{58}Ni at 20 MeV. In the scattering of ^{7}Li by ^{12}C, the validity of the formula is spoiled by the complexity of interactions which include target excitations. For the third-rank tensor analyzing powers for spin 3/2 particles a similar relationship is derived under the same assumption. The coupled-channel calculation clarifies why the relationship is valid approximately for the single-channel case but not for the four-channel case. From these results, the relationship seems to be useful in identifying effects of spin-dependent interactions directly from experimental data.

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DISCUSSION

MORAVCSIK: As I understand you keep only terms square and linear in F_{00}. If so, you should be able to check that assumption also on other types of observables. Have you done so?

TANIFUJI: No, I didn't try on other observables. For example, there is no data on iT_{31}, iT_{32} and iT_{33}, separately.

FICK: In any case the third rank analyzing powers T_{3q} are almost zero for ^{7}Li-^{58}Ni interaction at 20 MeV by dynamical reasons!