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2.20 Derivation of Dynamical Spin-Dependent Optical Potential by Páde Method

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In this note we will show an analytical representation of the dynamical optical potential generated from virtual-excitation process of a projectile in elastic scattering from 0^+ nuclei. We expect that such a work may be useful to understand polarization phenomena in elastic scatterings.

We take ^{7}Li as an example of a projectile in this work. The ground state($I=3/2^-$) and the first excited state($I'=1/2^-$) of ^{7}Li are assumed to be well described by α + t cluster model. We define the interaction $V(\mathbf{r},\boldsymbol{\rho})$ as the sum of the triton(t)-target nuclei and the alpha(α)-target nuclei central interactions (neglecting the spin-orbit interaction of the triton which generates a momentum-dependent optical potential, as shown in Ref. 1)). The radial coordinate vectors $\mathbf{r}$ and $\boldsymbol{\rho}$ are shown in Fig. 1 with angular momentum used later.

Expand the interaction $V(\mathbf{r},\boldsymbol{\rho})$ in multipoles as usual;

$$V(\mathbf{r},\boldsymbol{\rho}) = \sum_k v_k(r,\rho)\,(\mathbf{C}^{(k)}(\hat{\mathbf{r}})\cdot\mathbf{C}^{(k)}(\hat{\boldsymbol{\rho}})), \tag{1}$$

where $\mathbf{C}^{(k)}$ is a renormalized spherical harmonic operator. This interaction induces a transition to the intermediate channel $|L'(\ell's)I';JM)$ from the ground state channel $|L(\ell s)I;JM)$, where L(J) denotes the orbital(total) angular momentum in the scattering state and ℓ and s are the internal angular momentum and spin of the triton. The matrix element is calculated using eq.(1),

$$(L'(\ell's)I';JM\,|\,V\,|\,L(\ell s)I;JM = (-1)^{I'+J+L}\begin{Bmatrix} L & I & J\\ I & L & K\end{Bmatrix}(L'\|\mathbf{C}^{(k)}\|L)\,((\ell's)I'\|v_k\mathbf{C}^{(k)}\|(\ell s)I). \tag{2}$$

Then we calculate the dynamical optical potential, which depends on angular momentum L, as

$$(L(\ell s)I;JM\,|\,VGV\,|\,L(\ell s)I;JM) = \sum_{L'I'k_1k_2}(-1)^{I'-I+L'-L}\,\hat{L}\,(L0k_10|L'0)\,(L0k_20|L'0)\begin{Bmatrix} L' & I' & J\\ I & L & k_1\end{Bmatrix}$$
$$\times\begin{Bmatrix} L' & I' & J\\ I & L & k_2\end{Bmatrix}F_{k_1k_2}(r,r')\;G_{L'}(r,r'), \tag{3}$$

where G is a Green's function of the intermediate state. We obtain the right-hand side of eq.(3) by neglecting the I' dependence in G and by using

$$F_{k_1k_2}(r,r') = ((\ell s)I\,\|\,v_{k_1}(r,\rho)\mathbf{C}^{k_1}(\hat{\boldsymbol{\rho}})\,\|\,(\ell's)I')\,((\ell's)I'\|\,v_{k_2}(r,\rho)\mathbf{C}^{k_2}(\hat{\boldsymbol{\rho}})\,\|\,(\ell s)I). \tag{4}$$

Since the dynamical potential given by eq.(4) includes, in principle, all ranks of the potential, we must extract the potential of the desired rank using a sum rule for 6-j symbols. Thus we obtain the strength of the $V^{(K)}(\,[\mathbf{I}]^{(K)}\cdot\mathbf{R}^{(K)}\,)$-type potential, where $[\mathbf{I}]^{(K)}=[\mathbf{I}\times\cdots\times\mathbf{I}]^{(K)}$ and $\mathbf{R}^{(K)}$ is an operator of rank K composed of some of the radial operators $\mathbf{r}$, $\mathbf{L}$ and $\mathbf{P}$.

$$V_L^{(K)}(r,r') = \sum_{k_1k_2}(-1)^{2I+k_1+k_2+K}\,\hat{K}^2\hat{L}^2\begin{Bmatrix} k_1 & k_2 & K\\ I & I & I'\end{Bmatrix}\frac{F_{k_1k_2}(r,r')}{(I\|[\mathbf{I}]^{(K)}\|I)\,(L\|\mathbf{R}^{(K)}\|L)}$$
$$\times\sum_{L'}(L0k_10|L'0)\,(L0k_20|L'0)\begin{Bmatrix} k_1 & k_2 & K\\ L & L & L'\end{Bmatrix}G_{L'}(r,r'). \tag{5}$$

We have to treat the L' dependence of the Green's function carefully, because $V_L^{(K)}$ vanishes if this L' dependence is dropped. The L' is a value around L which is restricted by the 6-j symbol in eq.(5). This fact seems to suggest expanding the $G_{L'}$ in G_L.

$$G_{L'} = \sum_n G_L(D_{LL'}\,G_L)^n, \tag{6}$$

where

$$D_{LL'} = G_L^{-1} - G_{L'}^{-1} = \frac{h^2}{2\mu r^2}\,(L'-L)(L'+L+1). \tag{7}$$

Thus it may become easy to sum over L' in eq.(5) correctly. Although this approach requires the infinite series in $V_L^{(K)}$, it can be represented by a simple fractional expression by making use of Páde method[2], provided the adiabatic approximation is made for G_L(not for $G_{L'}$ itself),

$$G_L(r,r') = -\frac{1}{\varepsilon}\delta(r-r'). \tag{8}$$

For the ^{7}Li case the contribution to dynamical optical potential via the projectile virtual excitation to the first excited state is calculated by eq.(5), as follows

$$(\text{Central potential}) = -\frac{F22}{r^2}\,\frac{1}{\varepsilon}\,\frac{(1-Lz)[1+(L+1)z]}{[1-(2L-1)z][1+(2L+3)z]}, \tag{9}$$

and the following spin-dependent potentials;

$$(\text{Spin-orbit potential}) = \frac{9}{5}\,\frac{F22}{r^2}\,\frac{1}{\varepsilon}\,\frac{z(1-z)}{[1-(2L-1)z][1+(2L+3)z]}, \tag{10}$$

$$(T_R\text{ type tensor potential}) = \frac{1}{2}\,\frac{F22}{r^2}\,\frac{1}{\varepsilon}\,\frac{2+5z+(2L-1)(2L+5)z^2}{[1-(2L-1)z][1+(2L+3)z]}, \tag{11}$$

$$(\text{3rd-rank tensor potential}) = 2\,\frac{F22}{r^2}\,\frac{1}{\varepsilon}\,\frac{1-4z}{[1-(2L-1)z][1+(2L+3)z]}\,\frac{1}{L(L+1)(L^2+L-2)}, \tag{12}$$

where ε denotes the excitation energy of ^{7}Li and $z = \frac{\hbar^2}{2\mu r^2}\frac{1}{\varepsilon}$.

The above results show that the dynamical potential has a strong L dependence, but we can see that two of them, the spin-orbit and the 3rd rank tensor potentials, do not display L dependence in their numerators. Therefore their effects may become rapidly weaker than the other two potentials with increasing incident energy.

In elastic scattering at high energy the dynamical potentials are still effective for the central and T_R-type tensor potentials, but the spin-orbit and 3rd-rank tensor potentials which are predicted by the folding model must play a more important role than that caused by the dynamical process.

Since the present consideration may be general, it readily applies to other scatterings, for example, alpha elastic scattering. It may be interesting to compare such work with earlier results on dynamical polarization potentials[3].

References

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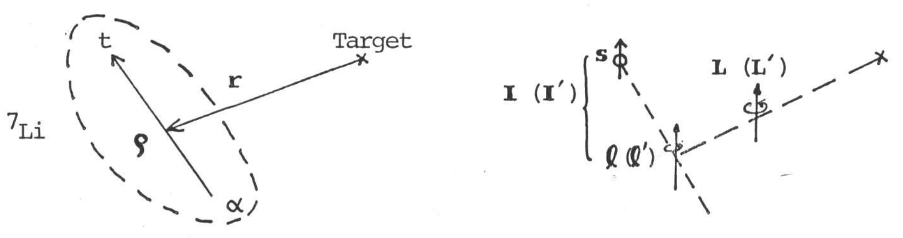

Fig. 1 Definition of radial distance and angular momentum vectors.