Proc. Sixth Int. Symp. Polar. Phenom. in Nucl. Phys., Osaka, 1985
J. Phys. Soc. Jpn. 55 (1986) Suppl. p. 852-853

3.25 Completeness of Observable Sets in pd Elastic Scattering

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Introduction. Progress in experimental techniques for polarized beams, targets and polarimeters has greatly increased the possibilities for polarization measurements. In particular, complete measurements (i.e., measurements that completely determine the scattering amplitudes in spin space) have become feasible even for reactions with rather complex spin structure, like $1/2 + 1 \rightarrow 1/2 + 1$, as is the case in pd elastic scattering. Given the new experimental capabilities, experimenters have to choose among different feasible sets of measurements the one that provides the most, and perhaps complete, information about the amplitudes in the most efficient way. In order to be able to make such decisions one has to study the relationship between observables and amplitudes. More specifically, one has to find a method to determine the number of independent observables in any given set. This is the purpose of this work which was prompted by two recent extensive experimental programs in pd elastic scattering, one at 10 MeV[1] (proton lab energy) and the other at 800 MeV.[2,3]

Method. For pd elastic scattering the matrix M of the scattering amplitudes is a 6-by-6 matrix. Assuming parity conservation and time reversal invariance, only 12 of the 36 amplitudes are independent.[4] Thus, a complete measurement includes at least 23 observables which determine all real parameters (but one generally undeterminable overall phase) of the 12 independent complex amplitudes.

The general polarization observable $C_{ij,kl}$ is related to the amplitudes by

$$C_{ij,kl} = \mathrm{Tr}[(\sigma_k \otimes \mathscr{P}_l)M(\sigma_i \otimes \mathscr{P}_j)M^\dagger]/\mathrm{Tr}(MM^\dagger) \qquad (1)$$

where σ_i and $\mathscr{P}_j$ (σ_k and $\mathscr{P}_l$) are the spin operators of the ingoing (outgoing) protons and deuterons, respectively.

Moravcsik et al.[5] have simplified the observable-amplitude relationship as much as possible by choosing appropriate spin operators. But even in the framework of this "optimal formalism" it is very difficult, if not impossible, to answer questions of practical relevance (like the one mentioned above) algebraically.

The method employed here was proposed by Simonius[6] 15 years ago. A computer program has been developed to investigate the Jacobian of Eq. (1) numerically. Arbitrary numbers are assigned to the amplitudes and the derivative of each observable of a given set is evaluated with respect to each of the 23 real parameters (magnitude squares and phases of the amplitudes). The rank N of the such obtained matrix (Jacobian) is then determined by checking row by row for linear independence. This procedure provides the first N independent observables in the given set. If N = 23, the set is complete (continuously complete, to be precise; there might still be discrete ambiguities).

Cartesian spin operators have been chosen because they are related in the simplest way to the experiments. The quantization axis for all four particles is taken to be normal to the scattering plane ("transversity formalism"). The M-matrix then assumes a particularly simple form, having just 12 non-zero elements.[4] The coordinate frame for each particle is chosen in accordance with the 1977 Ann Arbor Convention:[7] The common N-axis (quantization axis) is normal to the scattering plane, the L-axis is along the particle's momentum, and the S-axis is perpendicular to both N and L. Thus, in eq.(1) i,k=0,L,S,N and j,l=0,L,S,N,LL,SS,NN,LS,LN,SN.

Results and Conclusions. At 10 MeV proton lab energy a set of 19 observables has been obtained:[1] The unpolarized differential cross-section $d\sigma/d\Omega$, 5 analyzing powers (first order observables), 3 proton-to-proton and 10 proton-to-deuteron polarization transfer coefficients (second order observables). They are found to constitute an independent set. Furthermore, this set could be completed by measuring, for example, the four deuteron-to-deuteron transfer coefficients $C_{ON,ON}$, $C_{ON,OSS}$, $C_{ON,OLL}$ and $C_{ON,OLS}$ (all feasible measurements).

At 800 MeV, unpolarized target measurements[2] in the forward angular range include an independent set of 9 observables: $d\sigma/d\Omega$, 4 analyzing powers and 4 Wolfenstein parameters (proton-to-proton transfer coeffients). A large experimental program,[3] under way at the LAMPF High Resolution Spectrometer (HRS) has recently complemented this set with 15 second and third order observables in the small momentum transfer range, $-t$ between 0.03 and 0.17 $(GeV/c)^2$. In these experiments, polarized protons are scattered from a polarized deuteron target, with its polarization axis normal to the scattering plane and along the beam momentum, respectively, while the outgoing proton polarization is analyzed at the HRS focal plane. The resulting set of 24 observables is found to be complete and should provide the input for the first complete amplitude determination in pd elastic scattering.

Also at 800 MeV, but at larger momentum transfer, $-t$ up to about 0.8 $(GeV/c)^2$, a UCLA - U. of Iowa - Saclay collaboration will use a 1600 MeV polarized deuteron beam (same c.m. energy as 800 MeV proton lab energy) to bombard a polarized proton target and the recoiling proton polarization will be measured. This experiment will provide 27 new observables (complementing the 9 measurements from unpolarized target experiments mentioned above). This new set, together with $d\sigma/d\Omega$, is also found to be complete.

Generally, it seems not to be difficult to get a complete set of observables as long as a few of them connect proton with deuteron polarizations. This is illustrated by the following extreme example: By measuring only either proton or deuteron polarizations one could obtain 30 (linearly independent) observables. But only 20 of them turn out to be (non-linearly) independent. However, this set can be completed by adding just 3 "mixed" observables (connecting proton and deuteron polarization), for example the beam-target spin correlation coefficients $C_{NN,OO}$, $C_{SS,OO}$, and $C_{SL,OO}$.

The method, used here to analyze pd elastic scattering observable sets, could be applied to any other reaction or spin structure.

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