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# Transition matrix for reactions of spin structure $1 + 1/2 \rightarrow 1 + 1/2$ and first-rank observables in d-$^3$He scattering

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For d-$^3$He alastic scattering in the energy range $E_d$ = 10 -20 MeV an approximate equality takes place[1,2] between vector analyzing powers with the polarized beam ($iT_{11}$) and polarized target ($A_y^t$). In this work this similarity is analysed in terms of M-matrix.

Following ref.3, the M-matrix for reaction of spin structure $1 + 1/2 \rightarrow 1 + 1/2$ may be written in a factorised form $M = M_1 \otimes M_2$, here $M_1$ and $M_2$ are the M-matrices for the spin structures $1 + 0 \rightarrow 1 + 0$ and $0 + 1/2 \rightarrow 0 + 1/2$, respectively. The $M_1$ and $M_2$ matrices can be expanded as follows:

$$M_1 = a_o P_o + \sum_\lambda a_\lambda P_\lambda \,, \quad \lambda = k,\ kl; \quad k,l = x,y,z; \tag{1}$$

$$M_2 = \alpha_o \sigma_o + \sum_i \alpha_i \sigma_i \,, \quad i = x,y,z \,, \tag{2}$$

here $P_\lambda$ are cartesian spin-one tensors, $\sigma_i$ are Pauli matrices, $P_o$ and $\sigma_o$ are unit 3-by-3 and 2-by-2 matrices, respectively. The total 6-by-6 M-matrix is then represented as

$$M = A + \sum_i \alpha_i P_o \sigma_i + \sum_\lambda a_\lambda P_\lambda \sigma_o + \sum_{i\lambda} a_\lambda \alpha_i P_\lambda \sigma_i \,, \tag{3}$$

and its P-invariant form is as follows

$$\begin{aligned} M = {} & A + BP_o\sigma_y + CP_y\sigma_o + DP_{xz}\sigma_o + E(P_{xx} - P_{yy}) + FP_{zz}\sigma_o + \\ & GP_x\sigma_x + HP_z\sigma_x + KP_{xy}\sigma_x + LP_{yz}\sigma_x + JP_y\sigma_y + NP_{xz}\sigma_y + R(P_{xx} - P_{yy})\sigma_y + SP_{zz}\sigma_y + \\ & TP_x\sigma_z + UP_z\sigma_z + VP_{xy}\sigma_z + WP_{yz}\sigma_z \,. \end{aligned} \tag{4}$$

Inserting the explicit matrices $P_\lambda$, $\sigma_i$ into eq.(4), one obtains

$$M = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 & m_{10} & m_{11} & m_{12} \\ m_{13} & m_{14} & m_{15} & m_{16} & m_{17} & m_{18} \\ -m_{18} & m_{17} & -m_{16} & m_{15} & -m_{14} & m_{13} \\ m_{12} & -m_{11} & m_{10} & -m_9 & m_8 & -m_7 \\ -m_6 & m_5 & -m_4 & m_3 & -m_2 & m_1 \end{pmatrix} \tag{5}$$

with 18 independent matrix elements:

$m_1 = A + F + U,$
$m_2 = (1/\sqrt{2})(-iC + (3/2)D + T - (3/2)iW),$
$m_3 = 3E - (3/2)iV,$
$m_4 = -i(B + S) + H,$
$m_5 = (1/\sqrt{2})(G - (3/2)iL - J - (3/2)iN),$
$m_6 = -/i(R + K/2),$
$m_7 = (1/\sqrt{2})(iC + (3/2)D + T + (3/2)iW),$
$m_8 = A - 2F,$
$m_9 = (1/\sqrt{2})(-iC - (3/2)D + T + (3/2)iW),$
$m_{10} = (1/\sqrt{2})(G + (3/2)iL + J - (3/2)iN),$
$m_{11} = -i(B - 2S),$
$m_{12} = (1/\sqrt{2})(G + (3/2)iL - J + (3/2)iN),$
$m_{13} = 3E + (3/2)iV,$
$m_{14} = (1/\sqrt{2})(iC - (3/2D) + T - (3/2)iW),$
$m_{15} = A + F - U,$
$m_{16} = 3i(K/2 - R),$
$m_{17} = (1/\sqrt{2})(G - (3/2)iL + J + (3/2)iN),$
$m_{18} = -i(B + S) + H.$

For T-invariant elastic scattering, in the individual helicity or natural frames, it should be set in eq.(4)

$$D,\ T,\ H,\ L,\ V,\ N = 0\ . \tag{6}$$

That leads to additional reducing of the number of independent matrix elements:

$$m_2 = -m_7,\ m_3 = m_{13},\ m_4 = m_{18},\ m_5 = m_{12},\ m_9 = -m_{14},\ m_{10} = m_{17}, \tag{7}$$

and the elastic scattering M-matrix takes the form:

$$M = \begin{pmatrix} a & b & c & d & e & f \\ -b & g & h & i & j & e \\ c & -h & k & l & i & d \\ -d & i & -l & k & h & c \\ e & -j & i & -h & g & b \\ f & e & d & c & -b & a \end{pmatrix}, \quad \begin{matrix} a = m_1, & g = m_8, \\ b = m_2, & h = m_9, \\ c = m_3, & i = m_{10}, \\ d = m_4, & j = m_{11}, \\ e = m_5, & k = m_{15}, \\ f = m_6, & l = m_{16}. \end{matrix} \tag{8}$$

To reconstruct the M-matrix of eq.(8), 23 real parameters have to be found, i.e. 23 independent observables have to be measured.

Using M-matrix of eq.(8), the traces for vector analyzing powers $iT_{11}$ and $A_y^t$ are easy found:

$$iT_{11} = (\sqrt{3}/2)(\mathrm{Tr}\ MM^{+})^{-1}(8\mathrm{Re}AC^* - 4\mathrm{Re}CF^* + 6\mathrm{Re}UW^* - 12\mathrm{Re}CE^* + 8\mathrm{Re}BM^* - 6\mathrm{Re}KG^* - 12\mathrm{Re}MR^*), \tag{9}$$

$$A_y^t = (\mathrm{Tr}\ MM^{+})^{-1}(12\mathrm{Re}AB^* + 24\mathrm{Re}FS^* + 16\mathrm{Re}CM^* + 72\mathrm{Re}ER^*). \tag{10}$$

In general, eqs.(9) and (10) do not provide a ground to expect a similarity between $iT_{11}$ and $A_y^t$. On the other hand, as it follows from the calculations at low energies, the bilinear and more complex in spin-operator combinations are of little significance for <u>vector</u> analyzing powers[4,5]. Taking into account only linear in spin-operator terms in eqs.(9) and (10), a similarity between $iT_{11}$ and $A_y^t$ is obtainable by imposing a requirement

$$(\sqrt{3}/2)\ 8\mathrm{Re}AC^* \simeq 12\ \mathrm{Re}AB^*\ . \tag{11}$$

It seems naturally to involve, as a linear in spin-operator, the conventional spin-orbit interaction in the form $\lambda_\pi V_{so} f_{so}(r)\vec{L}\vec{s}$. Then taking into account the relation $\sigma_i = 2s_i$ for the case of spin-1/2, one obtains from eq.(11), if ReA, ImA $\neq 0$: $C \simeq \sqrt{3}\ B$, which may characterise the relative strength of spin-orbit interaction for deuterons and helions in d+$^3$He system. In the same time, the deviations from similarity of $iT_{11}$ and $A_y^t$ may be considered as an indication for possible contribution from interactions of more complex spin dependence. Therefore it is of interest to measure $iT_{11}$ and $A_y^t$ at higher energies where the role of these interactions is expected to be more significant.

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