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Comparison between Dirac Equation and Its Equivalent Schrödinger Equation for Inelastic Scattering

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Dirac equation has been used since some years to study proton elastic scattering at intermediate energies[1] (above 200MeV). This equation is :

$$\left[\vec{\alpha}.\vec{p} + \beta\left\{m + U_s(r)\right\} + U_v(\vec{r}) + V_c(\vec{r})\right]\Psi(\vec{r}) = E\Psi(\vec{r}) \qquad (1)$$

where $V_c(r)$ is the coulomb potential, $U_s(r)$ and $U_v(r)$, two complex potentials of which the real and the imaginary parts are approximated by a Woods-Saxon form-factor. For a given target and a given laboratory energy, the center of mass momentum k is fixed; we choose the reduced mass for m to define E with $E^2 = m^2 + k^2$. Using the decomposition into large and small components $\Psi(\vec{r}) = [F(\vec{r}), iG(\vec{r})]$, we get a system of two linear equations. The small components $G(\vec{r})$ can be eliminated. Using $F(\vec{r}) = D(\vec{r})^{1/2} f(\vec{r})$ where $D(\vec{r}) = E+m+U_s(\vec{r})-U_v(\vec{r})-V_c(\vec{r})$, we get the Schrödinger equation

$$\left\{\Delta - V_1(\vec{r}) - i\vec{\sigma}\vec{\nabla}V_2(\vec{r})\vec{\nabla} - k^2\right\} f(\vec{r}) = 0 \qquad (2)$$

$$V_1(\vec{r}) = \frac{3}{4}\left(\frac{\vec{\nabla}D}{D}\right)^2 - \frac{\Delta D}{2D} + \frac{E}{m}\left\{U_v(\vec{r}) + V_c(\vec{r})\right\} + U_s(r) + \frac{1}{2m}\left\{U_s^2(\vec{r}) - (U_v(\vec{r})+V_c(\vec{r}))^2\right\} \qquad (3)$$

$$V_2(\vec{r}) = \mathrm{Ln}\{D(\vec{r})\} \qquad (4)$$

If we consider the potentials $U_s(\vec{r})$ and $U_v(\vec{r})$ as describing the target and its excited states (vibrational or rotational models) and the wave functions $\Psi(\vec{r})$ as describing the incident proton and the target, we can derive a set of coupled equations from eq.(1). Their solution is described in another communication for a more general equation. Furthermore, if we neglect the variation of E with the channels in $D(\vec{r})$ and the mixing of the channels due to the factor $D(\vec{r})^{1/2}$ in the definition of $f(\vec{r})$, we get from eq.(2) a set of coupled equations which differs from the conventional ones only by the radial dependence of the potentials.

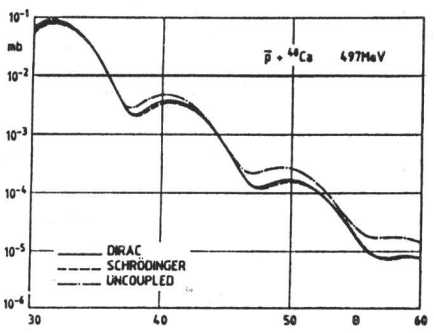

Fig. 1 : Elastic cross-section

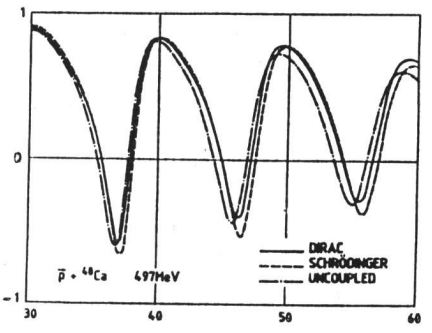

Fig. 2 : Elastic polarization

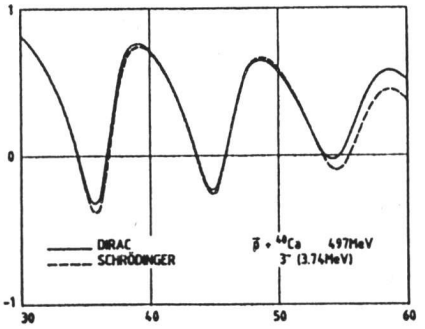

Fig. 3 : Inelastic analysing power

We have done calculations for the inelastic scattering of 497.5MeV protons on ^{40}Ca to the 3^- excited state at 3.7MeV, using the first set of parameters published in the analysis of elastic scattering by Clark et al[2]. Results obtained by using eq.(1) are quite the same as those obtained by using eq.(2). Effects of coupled channels are very small and DWBA[3] is enough for the inelastic scattering. However higher order effects show up in the elastic scattering as can be seen on Fig. 1 and 2. To show these effects without interference with other problems, we present here results obtained without charge (neutron scattering) between 30° and 60°. Below 30°, difference are too weak, above 60°, cross sections are too small. The conclusion is that eq.(2) slightly overestimates the coupling(the total cross-section for the 3^- is 2.527 mb with eq.(1) and 2.617 with eq.(2)). The largest difference at "experimental" angles is found in the elastic polarization. For the inelastic cross-section, the difference could not be seen on a figure. These two computations took almost the same time.

This comparison with a potential found in the literature does not prove that results are always similar. Indeed, in the study of inelastic scattering of 225MeV protons on ^{9}Be, we obtained a good fit to the analysing power with eq.(1) with a potential which gives worse results with eq.(2). We have still to see if slightly diffferent parameters would give a good fit with eq.(2) instead of eq.(1).

References

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