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5.3 Sum Rules Among Magnetic Moments of Octet Baryons

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Sum rules among magnetic moments of octet baryons have been discussed by several authors [1,2]. Sachs rule [1], for example, satisfies the experiment very well, which indicates the cancellation of nonstatic effects such as pionic corrections discussed by authors [3,4]. We have investigated baryon magnetic moments in the framework of SU(6) symmetry scheme, and had several sum rules among them. We have obtained good agreement with experiment, and shown that there are some cases where nonstatic effects cancell out.

[Type A] Dividing the octet members into two groups, we have the relation

$$\frac{1}{6}(\mu_p+\mu_n+\mu_{\Sigma^+}+\mu_{\Sigma^-}+\mu_{\Xi^\circ}+\mu_{\Xi^-}) = \frac{1}{2}(\mu_\Lambda+\mu_{\Sigma^\circ}) = \frac{1}{3}(\mu_u+\mu_d+\mu_s). \tag{1}$$

Following the simple SU(6) model ($\mu_u = -2\mu_d$ and $\mu_d = \mu_s$), the righthand side is just zero, while the experimental value of the lefthand side is 0.043 (n.m.). This relation is satisfied to a first order approximation. Here and hereafter, we use the experimental data referred in Ref. 1.

Similarly, we have

$$\frac{1}{3}(\mu_p+\mu_n+\mu_\Lambda) = \frac{1}{5}(\mu_{\Sigma^+}+\mu_{\Sigma^\circ}+\mu_{\Sigma^-}+\mu_{\Xi^-}+\mu_{\Xi^\circ}) = \frac{1}{3}(\mu_u+\mu_d+\mu_s). \tag{2}$$

Experimentally, the lefthand side is 0.089 n.m., approximately satisfied by the simple SU(6) model. By the same procedure, we obtain

$$\frac{1}{3}(\mu_p+\mu_{\Sigma^-}+\mu_{\Xi^\circ}) = \frac{1}{3}(\mu_n+\mu_{\Xi^-}+\mu_{\Sigma^+}) = \frac{1}{3}(\mu_u+\mu_d+\mu_s). \tag{3}$$

The experiment gives the values 0.177 n.m. and -0.091 n.m. for the first and the second expressions, respectively. Also, we have

$$\frac{1}{2}(\mu_p+\mu_n+\mu_{\Xi^-}+\mu_{\Xi^\circ}-2\mu_\Lambda) = \frac{1}{3}(\mu_u+\mu_d+\mu_s). \tag{4}$$

The lefthand side, empirically, turns out to be 0.083 n.m., close to the theoretical value (0.0) predicted by the simple SU(6) model.

[Type B] Taking the differences of magnetic moments, we have the relation

$$\frac{3}{5}(\mu_p-\mu_n) = \frac{3}{4}(\mu_{\Sigma^+}-\mu_{\Sigma^-}) = -3(\mu_{\Xi^\circ}-\mu_{\Xi^-}) = \mu_u - \mu_d\ , \tag{5}$$

from which we obtain

$$\mu_{\Sigma^+}-\mu_{\Sigma^-}+\ell(\mu_{\Xi^\circ}-\mu_{\Xi^-}) = \frac{1}{5}(4-\ell)(\mu_p-\mu_n)\ , \tag{6}$$

where ℓ is an arbitrary number. If ℓ is set to be one, this Eq. is written as follows ;

$$\mu_{\Sigma^+}-\mu_{\Sigma^-}+\mu_{\Xi^\circ}-\mu_{\Xi^-} = \frac{3}{5}(\mu_p - \mu_n),\ (2.78,\ 2.82). \tag{7}$$

The observed values lead that the lefthand side is 2.78 n.m. and the righthand side 2.82 n.m., in nice agreement with each others. The values in the parentheses after the Eq. represent the lefthand sum and the righthand one, respectively, in nuclear magnetons. Further, we comment on this sum rule. Sachs has derived [1]

$$\mu_{\Sigma^+}-\mu_{\Sigma^-}+\mu_{\Xi^\circ}-\mu_{\Xi^-} = 3(\mu_p+\mu_n),\ (2.78,\ 2.64)\ , \tag{8}$$

and Franklin has obtained [2,5]

$$\mu_{\Sigma+}-\mu_{\Sigma-}+\mu_{\Xi-}-\mu_{\Xi^o} = \mu_p-\mu_n , \quad (3.90, \ 4.70). \tag{9}$$

The latter relation is obtained, if ℓ is chosen to be -1 in Eq. (6). Though all these sum rules are derived by the SU(6) model, observed data satisfy Eqs. (7) and (8) very well, which means that nonstatic effects cancel out in Eqs.(7) and (8). Similarly, we obtain the relation

$$\mu_n-\mu_{\Xi^o}+\mu_p-\mu_{\Sigma^+} = \frac{3}{5}(\mu_{\Sigma-}-\mu_{\Xi-}) , \quad (-0.20, \ -0.19). \tag{10}$$

The observed value of the left-hand side is -0.20 n.m., so close to the right-hand side value -0.19 n.m. Also, we have

$$\mu_{\Xi-}-\mu_p+3(\mu_{\Sigma-}-\mu_n) = \frac{1}{5}(\mu_{\Xi^o}-\mu_{\Sigma+}) , \quad (-0.77, \ -0.72). \tag{11}$$

Empirically, the lefthand side is -0.77 n.m. and the righthand side is -0.72 n.m. Taking the sum of magnetic moments, we have the relations

$$\mu_p+\mu_n = \mu_u+\mu_d , \quad \text{and}$$
$$\mu_{\Xi^o}+\mu_{\Xi-} = -\frac{1}{3}(\mu_u+\mu_d) + \frac{8}{3}\mu_s . \tag{12}$$

Combining these Eqs. with next expressions

$$\mu_{\Sigma^o} = \frac{2}{3}(\mu_u+\mu_d) - \frac{1}{3}\mu_s , \quad \text{and} \tag{13}$$
$$\mu_\Lambda = \mu_s ,$$

We obtain

$$\mu_{\Xi^o}+\mu_{\Xi-} = -\frac{1}{3}(\mu_p+\mu_n) + \frac{8}{3}\mu_\Lambda , \quad \text{and}$$
$$\mu_{\Sigma^o} = \frac{2}{3}(\mu_p+\mu_n) - \frac{1}{3}\mu_\Lambda . \tag{14}$$

From these Eqs., we have

$$\mu_{\Xi^o}+\mu_{\Xi-}+\ell\mu_{\Sigma^o} = \frac{1}{3}(2\ell-1)(\mu_p+\mu_n) + \frac{1}{3}(8-\ell)\mu_\Lambda , \tag{15}$$

where ℓ is an arbitrary number. If ℓ is chosen to be zero, this Eq. is as follows ;

$$3(\mu_{\Xi^o}+\mu_{\Xi-}) + \mu_p + \mu_n = 8\mu_\Lambda , \quad (-4.94, \ -4.90). \tag{16}$$

The sum of empirical values in the lefthand side is -4.94 n.m., which is nicely related to the precisely measured value of Λ magnetic moment ($-0.613 \times 8 = -4.90$ n.m.).
Choosing ℓ to be -1 in Eq. (15), we obtain the sum rule derived by Franklin [2,6]

$$3\mu_\Lambda + \frac{1}{2}(\mu_{\Sigma+}+\mu_{\Sigma-}) - (\mu_{\Xi^o}+\mu_{\Xi-}) = \mu_p+\mu_n , \quad (0.76, \ 0.88). \tag{17}$$

If we utilize the experimental data referred in Ref. 2, the lefthand sum turns out to be 0.84 n.m., in better agreement with the righthand value of nucleon moments.

In summary, it is rather difficult to reproduce the absolute values of baryon magnetic moments, but there are several sum rules among them where the nonstatic effects cancell out and good agreement with experiment is obtained.

References

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