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Topological Structure of Hadronic Matter

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I discuss the structure of hadronic matter in terms of "tumbling" and topology and argue that quarks and gluons may not be needed at low energies for understanding the non-perturbative regime of QCD.

If quarks and gluons are the basic ingredients of the hadrons, as implied by QCD, how do they manifest themselves at low energies, say, $E < 1$ GeV? This is the question that preoccupies many nuclear physicists at the moment. In this talk, I would like to describe a scenario in which some topological concepts seem to play a key role, in a setting familiar to particle physicists and condensed matter physicists.

Let me start with the notion of "tumbling". It has been pointed out by Nambu[1)] that low-energy nuclear excitations in nuclei associated with pairing interaction may be governed by a sigma-model Lagrangian, "tumbling" down from a primary interaction at a higher-energy scale to a secondary interaction at a lower-energy scale. This resembles very much what happens in the technicolor theory of elementary particles[2)] in which at some high energies higher gauge symmetries are dynamically- and sequentially- broken down to lower gauge symmetries, arriving eventually at the standard model $SU(3)_c \times SU(2) \times U(1)$ of the strong and electroweak interactions which is then broken down at ~ 200 GeV to $SU(3)_c \times U(1)_{EM}$. This tumbling of interactions is assumed to be caused by growing interaction strength of the gauge couplings as energy is decreased[3)]. In the present case, we are not concerned with a gauge symmetry, but rather with a global symmetry of QCD, i.e., chiral $SU(N_f) \times SU(N_f)$ where N_f is the number of flavors (=3 for up, down and strange quarks). This symmetry is believed to spontaneously breakdown at energy scale $\Lambda \sim 1$ GeV to the diagonal subgroup $SU(N_f)_V$ (isospin or eightfold way), generating $(N_f{}^2-1)$ Goldstone bosons. The interaction that triggers this process may be called (following Nambu) "primary interaction". Below 1 GeV, the physics is described by a sort of sigma model consisting of Goldstone bosons, scalar mesons(σ) and low-lying baryons (Skyrmions, see below.) The Yukawa couplings in the sigma model induce strong forces by exchanging $\pi, \sigma,...$ which then trigger at some lower energy, say $\Lambda \sim 1$ MeV, "secondary interaction", causing condensates in the scalar channel (i.e. pairing), the pionic channel (i.e. pion condensates) etc. Nambu discusses

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how the IBM (interacting boson model), quasi supersymmetry and other low-energy phenomena could be described by means of this tumbling mechanism with its associated sigma-model Lagrangian.

It is quite remarkable that both the tumbling in gauge symmetry (technicolor) and the tumbling in chiral symmetry share the same σ-model structure (apart from the Higgs' mechanism in the former, absent in the latter). Another remarkable thing is that the secondary interaction as described above "knows" about the primary interaction, just as in the technicolor theory. Specifically, as noted by Nambu, the order parameter in the secondary interaction $< \sigma >$ is $\sim f_\pi$ ($\approx$ 100 MeV) which is also the order parameter in the primary interaction. This means that there may exist a close correlation between the primary interaction and the secondary interaction, not only in symmetries (i.e. σ-model) but also in dynamics. This is the basis on which my arguments given below are largely founded.

Let me first discuss an approach which in some sense relies on the tumbling idea, although the authors of the approach did not address the matter in that way. The question asked is: Can one construct a quark shell model for complex nuclei?[4)] Assume that $3A$ quarks move independently in the whole volume of the nucleus. Let the individual quarks occupy the various j-orbits in some central potential. Is it possible to obtain a reasonable description of the nucleus starting from a "quark soup" without ab initio constraint that AN_c quarks be clustered into A nucleons? This sounds like an impossible question to answer, but some initial attempts have been made recently. I will describe Talmi's recent analysis on the matter[5)].

Talmi assumes, as in nuclear shell model, a pairing interaction of the type acting in j-orbit of the quarks

$$V_j = -G_j \sum_{\alpha=1}^{N_c} A_\alpha^\dagger A_\alpha \tag{1}$$

where

$$A_\alpha|0>= \sum_{\beta,\gamma=1}^{N_c} \sum_{m=-j}^{j} \sum_{m_t=-1/2}^{1/2} (-1)^{j-m+\frac{1}{2}-m_t} \epsilon_{\alpha\beta\gamma} a_{\beta m m_t}^\dagger a_{\gamma -m -m_t}^\dagger |0>,$$

$$|0>= |T = J = 0> .$$

Here α, β, γ are color indices, N_c=3 and $a_{\beta m m_t}^\dagger$ is the creation operator for a quark in j-orbit, color index β, $m_j = m$, $m_t = \pm\frac{1}{2}$. Talmi then asks: Does the quark pairing interaction lead to a structure that resembles the shell model of the nucleons? He finds that indeed the lowest states in the model can be uniquely mapped onto states of nucleons in j-orbits, suggesting that symmetries in the primary interaction and the secondary interaction may be correlated. However there is a basic dynamical difference between the two due to confinement: in the quark shell-model, states of triplet quarks are correlated by their angular momenta in j-orbit whereas their radial wave functions extend over all volume of the nucleus, in contrast to those of confined quarks. This has large consequences on physical observables. For example, the spectroscopic factor for stripping a $1d_{3/2}$ proton or neutron from ^{40}Ca is predicted by the quark shell-model to be 0.084 while the nucleon shell-model predicts 4. (Experimentally it is even larger than 4.) This result clearly suggests that the tumbling must occur in a very intricate way and that confinement cannot be ignored even in the long wave-length limit. How to incorporate confinement in the quark shell-model is not known.

I now introduce an approach[6] that relies on topological concepts, which we claim is the most natural way (if not the only way) of treating the strong interactions at low energies. For this, return to the primary interaction, namely, the spontaneous symmetry breaking from chiral $SU(N_f) \times SU(N_f)$ to $SU(N_f)_V$. In the long wavelength limit, the Goldstone bosons are the only relevant degrees of freedom. Through topological transmutation, baryons emerge from the Goldstone bosons[7]. These are the Skyrmions[8]. Since the relevant energy scale is $\Lambda \sim 1$ GeV, however, the Goldstone bosons are not sufficient. Consistently with chiral symmetry and low-energy unitarity ($E \lesssim 1$ GeV), the vector mesons $\rho, \omega, ...$ must figure in the Skyrmion description as "hidden" gauge bosons[9]. Thus the physics at $E \lesssim 1$ GeV is again described by a σ-model of baryons (Skyrmions), pseudoscalar mesons (hidden scalar mesons) and vector mesons.

The connection to the fundamental theory, QCD, can be understood through the Cheshire Cat principle[6,10], which is best formulated in terms of the chiral bag[6]. Let us imagine inserting a (spherical) bubble of radius R at the center of a Skyrmion and filling it with N_c quarks. The question we ask is: Can this be done without destroying the baryonic structure of the Skyrmion? The answer is that it can be done by imposing a chiral boundary condition

$$-i\not{n}\psi = U_S\psi, \qquad U_S \equiv \exp i\lambda \cdot \pi\gamma_5/f_\pi \tag{2}$$

at the bubble surface and demanding that the quark spectrum including the negative energy sea be appropriately readjusted by the topological configuration of the punctured Skyrmion. In (1+1) dimensions, the chiral boundary condition plus an axial-flux matching condition are precisely the bosonization conditions and the physics is completely independent of the size of the bubble. In (3+1) dimensions, this Cheshire Cat property cannot be established rigorously. However recent works by A.D. Jackson and his collaborators[11] have shown that the Cheshire Cat principle holds remarkably well also in the four-dimensional situation.

It is easy to see what happens in (1+1) dimensions. Suppose that a bag wall is set up at $x = R$. Let us assume that the quarks are confined to the left of R. Imagine a quark travelling to the right. If the quark is massless and non-interacting, the right-moving quark above the Dirac sea cannot bounce off from the wall and move to the left above the Dirac sea without violating helicity. It can however plunge into the negative-energy sea and become a left-moving quark. One problem here is that one loses a quark charge if something does not take up the charge and move to the right. It is the meson field in $x > R$ that does this. In fact it is the punctured Skyrmion that carries exactly the missing charge. In (1+1) dimensions, the charge carried by the Skyrmion is $\theta(R)/\pi$ where θ is the profile function of the Skyrmion, which goes to π if the wall is moved to the extreme left (i.e. if the bag is shrunk to a point.) In (3+1) dimensions, it is somewhat more complicated but a qualitatively similar phenomenon occurs there too. One simple way of understanding this phenomenon is as follows. The boundary condition (2) is a twisted boundary condition, the twist corresponding to the chiral angle θ (from the hedgehog $\vec{I}\cdot\vec{\pi}/f_\pi \to \theta\vec{\tau}\cdot\hat{r}$). Let us untwist it by a local rotation, so that Eq.(2) becomes

$$-i\not{n}\psi = \psi.$$

However the rotation induces an axial gauge field which couples to the quark at the surface. Such couplings induce anomalies. In this case the axial gauge coupling induces an anomaly in the vector current. Consequently, the vector current associated with the quark charge will be non-conserved. Thus the leakage; and the topology insures that the amount of the charge leaked out be carried by the Skyrmion. N_c quarks make up a baryon and a leakage of N_c quarks makes the Skyrmion a baryon number $B = 1$ object.

I believe that the boundary condition (2) is essential in endowing a topological structure to the baryon. An effective model of quarks coupled to the chiral fields need not acquire a topological quantum number when the quarks are integrated out. Without (2) imposed at the origin, the topological charge obtained in the usual Goldstone-Wilczek method[12] just dwindles away and cannot be identified with a fermion or baryon number[13].

The baryon number is a topological invariant, but what about other quantities like the energy, magnetic moments and other static and dynamic observables of the baryons? They are not obviously connected with topology, so one cannot a priori expect a Cheshire Cat property for them. However there is a very strong indication[11] that no physical observables depend in any significant way on the bubble radius R or on any details of the separation into the quark sector and the Skyrmion sector. There is a further indication that this property continues to hold even when strange quarks are introduced[14].

The Skyrmion can then be viewed as a chiral bag shrunk to a point. The resulting effective theory is then given in terms of chiral fields, from which baryons emerge as topological solitons. This theory is consistent with QCD in the sense that the chiral fields (point like) can be viewed as bits of flux tubes on loops figuring naturally in gluodynamics of QCD[15]. One can expect that the Regge trajectories of the baryons would also emerge from the Skyrmion Lagrangian. Given the σ-model with baryons and mesons, the secondary interaction is easy to understand as explained in ref.6. The question can arise however as to how far up in energy one can push the Skyrmion-type Lagrangian. The answer essentially lies in the chiral scale $\Lambda \sim 1$ GeV. Up to this scale, we expect that the theory could work satisfactorily. Up to date, there is no experimental indication that it does not: No signatures of explicit "quark presence" in nuclei have been exhibited. In fact, one can use the σ-model Lagrangian beyond the nuclear matter density and describe the equation of state of dense hadronic matter relevant in supernova explosion and heavy ion collisions[16]. Chiral constraints play an important role there.

It is often claimed that as the nucleons in nuclei overlap, antisymmetrization between quarks lodged in two or more nucleons must become important. Such considerations often predict significant deviations from the picture of the nucleons that are properly antisymmetrized. For simplicity, focus on two nucleons. Let us consider them as two partially overlapping chiral bags. Assuming the bag size to be small, the overlapping then involves two fractionized bits of Skyrmions. They need to satisfy neither fermi statistics nor bose statistics. (In some (2+1) dimensional σ models, the Skyrmions can satisfy even fractional statistics[17].) In fact, the only constraint is that the two nucleons as a whole (two Skyrmions) be antisymmetrized. The prediction in this picture is then that to the order we consider (i.e. N_c expansion) there should be no effects of quark-antisymmetrization.

One of the most remarkable features of the Cheshire Cat phenomenon is that one may be able to describe chiral phase transition without quarks and gluons[19,11]. If one were to describe asymptotic freedom with a σ-model type Lagrangian, one would have to introduce an infinite number of meson fields[18]. Thus if the Wigner phase were also in an asymptotically free regime, it would be meaningless to talk about chiral phase transition in terms of a tractable chiral Lagrangian. But this may not be so. In fact it seems reasonable that there are bound states, realized in Wigner mode, after the phase transition. If this is so, describing chiral phase transition in terms of Skyrmions seems highly sensible. In this picture, the phase transition may be viewed as a change of topological mappings. In the dilute phase where the Skyrmions are widely separated, the individual Skyrmion is best described by a mapping of hedgehog type (from a physical space to a target manifold which is an internal symmetry space). In the dense phase beyond ρ_c (critical density), the Skyrmion is described by an identity mapping. The change of maps is associated with the change of order parameter, say, the σ condensate

$< \sigma >$ which is non-zero in the former and zero in the latter. Physically, the latter may be viewed as the Wigner phase. This geometrical interpretation, due initially to Manton[19] and further developed by Jackson[11], is in qualitative agreement with the QCD picture. The quantitative structure of the process (such as critical density etc.) will, however, depend on the dynamical contents of the Lagrangian used.

It is remarkable that so much of the hadronic matter can be described in one coherent scheme based on chiral symmetry. What appeals to me in this way of looking at things is that the fundamental properties of the nucleons (baryons in general), of the nuclei and of hadronic matter under extreme conditions are interrelated through symmetries and associated topological considerations. In this connection, I might mention that the Wess-Zumino term in the Skyrmion picture (the term in which chiral anomalies are encoded) plays an equally important role in the hyperon structure[20] and in nuclear exchange currents[21] and in other areas of physics (e.g. strings.) There is no need at low energies of explicit presence of quarks and gluons even if QCD is the ultimate theory.

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