Proc. Fifth Int. Conf. Clustering Aspects in Nucl. and Subnucl. Systems, Kyoto, 1988
J. Phys. Soc. Jpn. 58 (1989) Suppl. p. 626–632

Angular Momentum Quasiparticle Approach to Cluster States in Nuclei[1]

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Quasi-Free $(\vec{p}, 2p)$ reactions at medium energies are considered. In this context, a simple and rather direct method to extract informations about cluster correlations in nuclei is presented. It is shown that, if the initial nucleus has a nonvanishing spin, the effective polarization for a knock-out leading to a given final state can be expressed as a linear combination of effective polarizations defined in the basis of an angular momentum quasiparticle approach to cluster states in nuclei. Analyzing the effects of nuclear medium on the effective polarization, some informations about nuclear structure — particularly those associated with orbital angular momentum and/or spin cluster correlations in nuclei — may be obtained. The results of recent calculations of the effective polarizations for the reactions $^6Li(\vec{p}, 2p)\,^5He$ and $^{14}N(\vec{p}, 2p)\,^{13}C$ at 320 MeV are shown and compared to previous results obtained for the single-particle $[jj]$ shell model.

Numerous experimental and theoretical investigations on quasi-free reactions, performed in the last three decades, have resulted in a large body of informations about binding energies and momentum distributions of nuclear nucleons, widths of one hole states and spin-orbit splittings of nuclear shells[1]. This considerable amount of work has established, in a very consistent way, the basic conceptual framework of quasi-free processes, i.e. the understanding of the reaction mechanism, the limitations of the Distorted Wave Impulse Approximation (DWIA) and of the Factorization Approximation, the role of the initial states interactions and the properties of the states of the residual nucleus.

Quasi-free nucleon-nucleus scattering at medium energy emerged from these studies as one of the most important tools for investigating the single-particle properties of a nucleus (its shell structure), in particular for the most strongly bound states, and effects of the nuclear environment on the short range structure of the bound nucleons.

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[1]Supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, Brazil) and by the Fundação de Amparo à Pesquisa no Estado do Rio Grande do Sul (FAPERGS, Brazil)
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In this note we consider quasi-free reactions and the cluster model description of nuclear states. We shall see that if the initial nucleus has a nonvanishing spin, we can express the effective polarization for a knock-out leading to a given final state as a linear combination of effective polarizations defined in the basis of an angular momentum quasiparticle approach to cluster states in nuclei, weighted by distorted momentum distributions.

The annihilation operator for an A nucleon state, defined in terms of a single-particle $[jm]$ shell-model basis, is given by

$$a_{jm} = \sum_{1\ldots A-1} \mid 1,\ldots,A-1 >< jm; 1,\ldots,A-1 \mid, \tag{1}$$

with $\eta = [j_\eta m_\eta]\, for\, \eta = 1 \ldots A-1$.

For simplicity we assume that the motion of the A nucleons is dynamically correlated in such a way that their behaviour is described by a two-cluster model (the extension of the following discussion to more than two clusters can be done in a very easy and direct way). Furthermore, we require that this model be independent of the total center-of-mass motion (this avoids non-physical (spurious) collective excitations of our system). To garantee translational invariance, we introduce a more consistent representation, namely a cluster model basis defined in terms of internal and relative motion coordinates (see, for example, reference [2] for the usual definitions of these new coordinates). In this representation, the internal motions of the clusters 1 and 2 are characterized by the quantum numbers j_{ρ_κ} and m_{ρ_κ} (with the κ's ranging from 1 to ξ (= number of internal coordinates) for the cluster 1 and from $\xi + 1$ to $A - 3$ for the cluster 2 (remember that the new coordinates are not linearly independent)) whereas the relative motion is characterized by the quantum numbers j_R and m_R. In this model the wave function of the initial state is then given by

$$\begin{aligned} \mid \Psi_i > \;=\; & \sum_{j_{\rho_i} m_{\rho_i}} \sum_{j_{R_i} m_{R_i}} C(j_{\rho_i} m_{\rho_i}; j_{R_i} m_{R_i} \mid j_i m_i) \\ & \cdot \mid \psi_{j_{\rho_i} m_{\rho_i}}[\varphi(1)\varphi(2)] \chi_{j_{R_i} m_{R_i}}(R) >, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mid \psi_{j_{\rho_i} m_{\rho_i}}[\varphi(1)\varphi(2)] > \;=\; & \sum_{j_{\rho_a} m_{\rho_a}} \sum_{j_{\rho_b} m_{\rho_b}} C(j_{\rho_a} m_{\rho_a}; j_{\rho_b} m_{\rho_b} \mid j_{\rho_i} m_{\rho_i}) \\ & \cdot \mid \varphi_{j_{\rho_a} m_{\rho_a}}(\rho, \rho_1 \ldots \rho_\xi) \varphi_{j_{\rho_b} m_{\rho_b}}(\rho_{\xi+1} \ldots \rho_{A-3}) >; \end{aligned} \tag{3}$$

in this expression the symbols $j_{\rho_a} m_{\rho_a}$ and $j_{\rho_b} m_{\rho_b}$ characterize the total angular momenta and their projections for the clusters 1 and 2, respectively. The final state wave function reads

$$\begin{aligned} \mid \Psi_f > \;=\; & \sum_{j_{\rho_f} m_{\rho_f}} \sum_{j_{R_f} m_{R_f}} C(j_{\rho_f} m_{\rho_f}; j_{R_f} m_{R_f} \mid j_f m_f) \\ & \cdot \mid \psi_{j_{\rho_f} m_{\rho_f}}[\varphi(1')\varphi(2')] \chi_{j_{R_f} m_{R_f}}(R') >; \end{aligned} \tag{4}$$

above $\mid \psi_{j_{\rho_f} m_{\rho_f}} >$ can be expanded similarly as in Eq. (3).

In this new basis the a_{jm} operator (Eq.(1)) may be written accordingly as

$$a_{jm} = \sum [u_{jm} M^{jm}(\rho ... R') \mid \rho'_1 ... \rho'_\xi ; \rho'_{\xi+1} ... \rho'_{A-s} ; R' >< \rho, \rho_1 ... \rho_\xi ; \rho_{\xi+1} ... \rho_{A-s} ; R \mid + v_{jm} M_{j\tilde{m}}(\rho ... R') \mid \rho', \rho'_1 ... \rho'_\xi ; \rho'_{\xi+1} ... \rho'_{A-s} ; R' >< \rho_1 ... \rho_\xi ; \rho_{\xi+1} ... \rho_{A-s} ; R \mid]. \tag{5}$$

In this expression, the sum extends from $\rho, \rho_1 ... \rho_{A-s}, R$ to $\rho'_1 ... \rho'_{A-s}, R'$ and the ρ's and R's represent short notations for the quantum numbers $j_{\rho_\kappa} m_{\rho_\kappa}$, $j_R m_R$,etc. The matrices M^{jm} are the transformation coefficients from the old basis to the new one; finally, the primes are remainders of the fact that the internal coordinates are not linearly independent whereas the $\sim$ indicate that the magnetic quantum numbers m_{ρ_κ} and M_R should be replaced respectively by $-m_{\rho_\kappa}$ and $-M_R$.

Inserting in this expression the unit operator defined in terms of the new coordinates and assuming, furthermore, that the A nucleon Hamiltoninan is separable in these coordinates, we have

$$a_{jm} = u_{jm} \Lambda_{jm} + v_{jm} \Lambda^\dagger_{jm}, \tag{6}$$

where

$$\Lambda_{jm} = \sum_{j_\rho m_\rho} \sum_{j_R m_R} C(j_\rho m_\rho ; j_R m_R \mid jm) \alpha_{j_\rho m_\rho} \otimes W_{j_R m_R}. \tag{7}$$

In this expression, $\alpha_{j_\rho m_\rho}$ represent annihilation operators in $\rho - space$; the operator $W_{j_R m_R}$ is defined by the angular momentum expansion of the annihilation and creation operators $W_{j_{R_i} m_{R_i}}$ and $W^\dagger_{j_{R_f} m_{R_f}}$. From these equations we see that we have expanded the one-particle annihilation operator a_{jm} via a set of new operators, i. e. $\alpha_{j_\rho m_\rho}$, $\alpha^\dagger_{j_\rho m_\rho}$, $W_{j_{R_i} m_{R_i}}$, and $W^\dagger_{j_{R_f} m_{R_f}}$ which act in this model on the internal and relative motion vectorial states, respectively, annihilating or creating states with definite angular momenta and projections. The resulting Hamiltonian, expressed in terms of this set, describes in the initial and final states the dynamical behaviour of one R- and n and $n-1$ ($n = 0, 1, ...$) ρ-quasiparticles. We could associate the so called R-quasiparticles with collective excitations in the nucleus. In this picture, the destruction of a nucleon in the conventional single- particle description could be interpreted, when translated to our collective cluster model, as the destruction of an internal ρ-quasiparticle acompanied by a change in the collective excitation of the system. Since, in this picture the spin character is carried over only by the ρ-quasiparticles, the ρ-operators are fermion-like and the corresponding R-quasiparticles are boson-like objects. In Eq.(6), the coefficients u_{jm} and v_{jm} are assumed to be real and spherically symmetric. The condition $u^2_{jm} + v^2_{jm} = 1$ ensures, in particular, that the fermion-like quasiparticle annihilation and creation operators, Λ_{jm} and $\Lambda^\dagger_{jm}$, respectively, satisfy the anticomutation relation $[\Lambda_\alpha, \Lambda^\dagger_\beta]_+ = \delta_{\alpha\beta}$ as do the original single-particle operators a_{jm} and $a^\dagger_{jm}$.

Combining these definitions with the definition of the effective polarization [1] and using the orthogonality properties for the Clebsch-Gordan coefficients and the Wigner-Eckart theorem we obtain an expression for the effective polarization in the form (see also[3])

$$P_{eff}(\vec{k}) = \frac{\sum_{j_\rho} |\gamma_{j_\rho}|^2 P_{j_\rho}(\vec{k}) \, |G'_{j_\rho}(\vec{k})|^2}{\sum_{j_\rho} |\gamma_{j_\rho}|^2 |G'_{j_\rho}(\vec{k})|^2}. \tag{8}$$

In this expression γ_{j_ρ} denote reduction parameters (reduced matrix elements) obtained from direct application of the Wigner-Eckart theorem to the matrix elements for the knocking-out of a ρ-quasiparticle ; $G'_{j_\rho}(\vec{k})$ is the distorted momentum amplitude defined in the context of the cluster model. As the angular momenta and projections are coupled in the form: $\vec{j}_i = \vec{j}_{\rho_i} + \vec{j}_{R_i}$, $m_i = m_{\rho_i} + m_{R_i}$, $\vec{j}_f = \vec{j}_{\rho_f} + \vec{j}_{R_f}$, $m_f = m_{\rho_f} + m_{R_f}$, and since $\vec{j} = \vec{j}_i - \vec{j}_f$ together with $m = m_i - m_f$, we have $\vec{j} = \vec{j}_\rho + \vec{j}_R$, $m = m_\rho + m_R$, with $\vec{j}_R = \vec{j}_{R_i} - \vec{j}_{R_f}$ and $m_R = m_{R_i} - m_{R_f}$.

The dependence of the effective polarization on the γ_{j_ρ} coefficient is very sensitive to the nuclear structure and may provide, when compared to experiments, rather direct informations on cluster correlations in nuclei.

At this point it would be interesting to compare typical results of the calculations of the effective polarizations for the reactions $^{14}N(\vec{p},2p)^{13}C$ and $^{6}Li(\vec{p},2p)^{5}He$ (see Figure 1). Recent predictions from a $[core{+}deuteron]_{L=0,S=1}$ cluster model are compared with previous findings obtained for the single-particle shell model with $[jj]$ coupling. Evidently, the effective polarizations for the two models show striking differences: they differ both in sign and size; both effects are fairly stable against reasonable parameter variations.

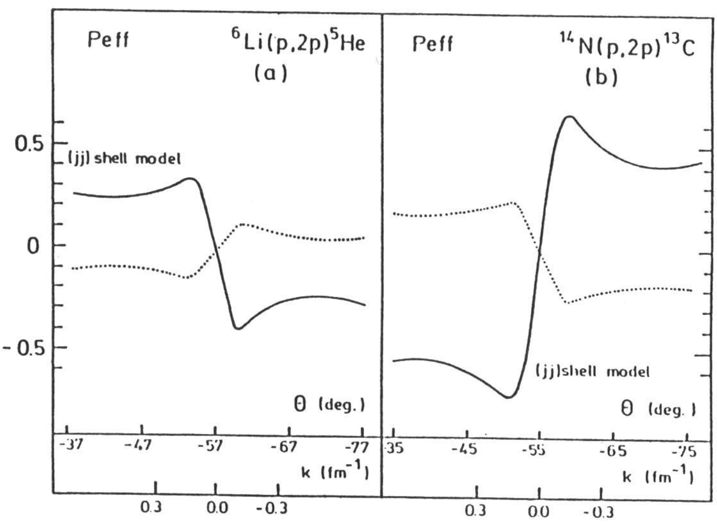

Fig. 1. Effective polarizations calculated for the nuclear reactions $^{6}Li(\vec{p},2p)^{5}He$ (a) and $^{14}N(\vec{p},2p)^{13}C$ (b) at a lab energy of 320 MeV. Above, the full lines correspond to the shell model, the dotted lines to the two-body cluster model; $\vec{k}$ is the linear momentum of the knocked-out proton and Θ is the scattering angle.

In the $[jj]$ coupled shell model the effective polarization is caused by the correlations of the spin and angular momentum of the proton which is knocked-out from the initial nucleus, combined with distortion effects in the selected asymmetric geometry of the experiment. For the cluster model, one could expect, "a priori" , that the effective polarizations should be equal to zero since it seems, at a first glance, that there are no correlations of this type in the initial nuclei because the internal and relative motion wave functions of the deuteron surrouding the core represent both $L = 0$ states. In fact, this is not so as shown in Figure 1. The effective polarization in the ^6Li case is about $-1/2$ times the corresponding one for the ^{14}N. This result is in agreement with the anticipated trend[1] for the effective polarizations of the $1p_{3/2}$ and $1p_{1/2}$ single-particle states in the case of the shell model with $[jj]$ coupling. We show, in the following, that these results could be obtained, in the case of the cluster model, directly from Eq. (8).

The differences between the results of the shell and cluster models are easy to understand (see reference[5]). Because of the deuteron spin wave function with $S = 1$, the spin of the knocked-out proton is parallel to the spin of the remaining valence neutron. The momentum of this neutron has, however, a tendency to be opposite to that of the knocked-out proton, because both momenta are anticorrelated in the deuteron. Using the distortion arguments of reference[4] but now applied to the final nucleus, one finds again that the remaining neutron is effectively polarized but oppositely to the knocked-out proton in the shell model, because of its opposite internal momentum. This polarization carries over to the knocked-out proton through the mentioned $S = 1$ correlation, which explains the difference in sign of the effective polarization in the two models. The quantitative difference of the polarization is caused by the fact that in the cluster model the momentum of the remaining valence neutron is not exactly opposite to the one of the knocked-out proton but is smeared out by the center of mass motion of the deuteron.

These results may be obtained from Eq.(8) in the following way. We construct an "effective internal spin-orbit coupling" of the type $\vec{\ell}_\rho \cdot \vec{s}_\rho$ for the ρ-quasiparticles where $\vec{\ell}_\rho$ and $\vec{s}_\rho$ characterize, respectively, their (conserved, in our model) internal angular momentum and spin and define $\vec{j}_\rho = \vec{\ell}_\rho + \vec{s}_\rho$ and $m_{j_\rho} = m_{\ell_\rho} + m_{s_\rho}$. In the case in which we could associate to a given nucleus only one value of $\vec{j}_\rho$ this would correspond, in our model, to a "pure internal configuration" with $j_\rho = \ell_\rho + 1/2$ or with $j_\rho = \ell_\rho - 1/2$ for that nucleus. For a nucleus characterized by "internal configuration mixing" with a fixed value of ℓ_ρ, from Eq.(8) we have

$$P(\vec{k}) = \frac{P_{\ell_\rho+1/2}(\vec{k}) + \eta_\rho P_{\ell_\rho-1/2}(\vec{k})}{1+\eta_\rho}, \tag{9}$$

where

$$\eta_\rho = \frac{|\gamma_{\ell_\rho-1/2}|^2}{|\gamma_{\ell_\rho+1/2}|}. \tag{10}$$

From the definitions of the ρ-quasiparticle operators, of the ρ-vacuum, and of the γ_{j_ρ} coefficients, we have, for $j_i = 0$ and taking fixed values of $j_{\rho_f} = j_\rho = \ell \pm 1/2$, and $m_{\rho_f} = -m_\rho$, from Eq.(10), with $j_\pm = \ell_\rho \pm 1/2$ and $m_\pm = m_{\ell_\rho \pm 1/2}$,

$$\eta_\rho = \frac{C^2(j_+, j_+; m_+, -m_+ \mid 00)}{C^2(j_-, j_-; m_-, -m_- \mid 00)} = \frac{\ell_\rho}{\ell_\rho + 1}. \tag{11}$$

In case $P(\vec{k}) = 0$, from (9) and (11) we obtain

$$P_{\ell-1/2}(\vec{k}) = -\frac{\ell+1}{\ell} P_{\ell+1/2}(\vec{k}). \tag{12}$$

Combining these definitions we obtain for the effective polarization, in the general case of a state with "internal configuration mixing",

$$P(\vec{k}) = \frac{(\eta_\rho - 1)}{(\ell_\rho + 1) + \eta_\rho \ell_\rho} \cdot P_{\ell_\rho -1/2}(\vec{k}). \tag{13}$$

A change of basis from the $j_\rho m_\rho$ basis to the $L_\rho S_\rho$ one with $\vec{L}_\rho = \sum \vec{\ell}_\rho$ and $\vec{S}_\rho = \sum \vec{s}_\rho$ gives for the knock-out of a ρ-quasiparticle from the deuteron cluster in the model [core+deuteron] for a fixed value of L_ρ and S_ρ

$$|\gamma_{j_\rho}|^2 = [(2j+1)(2j_\rho+1)(2S_\rho+1)(2L_\rho+1)] \left\{ \begin{array}{ccc} 1/2 & \ell_\rho & j \\ 1/2 & \ell_\rho & j_\rho \\ S & L_\rho & j_i \end{array} \right\}^2, \tag{14}$$

in which the { } represent Wigner $9j$-coefficients. For the state with $L_\rho = 0$ and $S_\rho = 1$ and from the definition of the η_ρ parameters, expression (14) gives $\eta_\rho = 2$ for $j = 3/2$ and $\eta_\rho = 1/2$ for $j = 1/2$. When combined with Eq. (13) these results show that the sign of the effective polarizations in the $[core + deuteron]_{L=0,S=1}$ cluster model for the reactions $^6Li(\vec{p}, 2p)\,^5He$ and $^{14}N(\vec{p}, 2p)\,^{13}C$ should indeed be different from the corresponding one in the single- particle model. In this case the experimental information on the quasi-free asymmetries might help to shed light on the two models for the description of angular momentum and spin correlations in nuclei.

Acknowledgments

The author is very grateful to Prof. Dr. M. Dillig (Institut für Theoretische Physik III, Erlangen, Federal Republic of Germany) for some important comments and also for his kind hospitality.

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