Proc. Fifth Int. Conf. Clustering Aspects in Nucl. and Subnucl. Systems, Kyoto, 1988
J. Phys. Soc. Jpn. 58 (1989) Suppl. p. 639–645

ENERGY DEPENDENCE OF ASTROPHYSICAL S-FACTOR FOR RADIATIVE CHARGED PARTICLE REACTIONS

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The energy dependence of astrophysical S-factors for the ^{4}He(^{3}H, γ)^{7}Li, ^{7}Be(p, γ)^{8}B, ^{20}Ne(p, γ)^{21}Na and ^{12}C(α, γ)^{16}O reactions is studied theoretically. The observed S-values at higher energies are extrapolated to zero energy by using the inferred theoretical energy dependence, which are applied to several astrophysical problems.

§1. Introduction

There are several key reactions for the cosmological creation of elements. The radiative alpha capture reaction ^{4}He(^{3}H, γ)^{7}Li is one of these which creates primordial ^{7}Li. Although a homogeneous big-bang model[1)] identifies only the light nuclei ^{2}H, ^{3}He, ^{4}He and ^{7}Li to be primordial, an inhomogeneous model[2-5)] predicts that even heavy elements like carbon, nitrogen, silicon, etc., also should have been produced in the early universe[6)] via

$$^7\mathrm{Li}(n, \gamma)^8\mathrm{Li}(\alpha, n)^{11}\mathrm{B}(n, \gamma)^{12}\mathrm{B}(\beta^- \nu)^{12}\mathrm{C}\,(n, \gamma)^{13}\mathrm{C}(n, \gamma)\ \ldots$$

The ^{4}He(^{3}H, γ)^{7}Li reaction takes a key to test the cosmology, homogeneous or inhomogeneous. Three independent experimental groups[7-9)], however, observed quite different energy dependence of the reaction rate for this key reaction. The calculated ^{7}Li abundance in the big-bang model hence changes by nearly a factor of two depending on which data are used.[10,11)] A reliable theoretical extrapolation of the observed data to the astrophysical low energy E_{cm} = 100 keV is needed.

The ^{7}Be(p, γ)^{8}B reaction is another key reaction of astrophysical interest in the missing solar neutrino problem.[12)] This problem is still unresolved, leaving a factor of three discrepancy between the theoretical[13)] and observed[14)] neutrino counting rates.* If the standard solar model is a good approximation to the true sun, the major ambiguity of the theoretical prediction

* The most recent run of the detection[19)] in ^{37}Cl seems to indicate 4.0 ± 0.3 SNU (1 solar neutrino unit = 1 capture/sec per 10^{36} detector atoms) which is more than two times larger than the previous averaged values[14)] 2.0 ± 0.3 SNU. The reason for this discrepancy is not clear. Theoretical prediction[13)] is 7.9 ± 2.6 SNU.

arises from the nuclear reaction rate of ^{7}Be(p, γ)^{8}B. The source of the ambiguity is twofold[11,15]; the first is the scattered different experimental data even after correcting the normalization based on the finding of Filippone et al.[16] on the ^{7}Li(d, p)^{8}Li reaction rate, and the second is the different prediction of energy dependence with[17] and without[18] the d-wave contribution as well as the s-wave contribution to the S-factor. The second uncertainty results in a 15% different theoretical neutrino counting rate and must be removed theoretically.

In novae and accreting white dwarfs is presumed to take place the hot CNO-cycle (or rapid-proton process).[20] One of the key reactions here is ^{19}Ne(p, γ)^{20}Na which triggers a leakage from the cycle. Kubono et al.[21] have recently found several excited states of ^{20}Na just above the proton separation threshold. Their finding suggests a hundred times stronger ^{19}Ne(p, γ)^{20}Na reaction rate than the beta decay ^{19}Ne($\beta^+\nu$)^{19}F rate at T = 3x10^8 K. Once the leakage takes place actively, ^{20}Na decays to ^{20}Ne and the next proton capture ^{20}Ne(p, γ)^{21}Na follows to start Ne-Na cycle. Therefore, the ^{20}Ne(p, γ)^{21}Na reaction rate, which accelerates or deaccelerates the Ne-Na cycle, is to be known precisely. ^{21}Na has several bound states contributing to the radiative capture and each transition rate indicates quite different energy dependence. A reliable extrapolation to the astrophysical energy region is again required here.

The most important nuclear reaction in massive stars is ^{12}C(α, γ)^{16}O [ref. 22]. Although several independent experiments indicate different astrophysical S-factors at higher energies 1.0 MeV < E_{cm}, the main difficulty in the extrapolation arises from strong energy variation of the high-energy-tail of subthreshold 1$^-$ state of ^{16}O. Theoretical analysis of the energy dependence of the S-factor has a chance to give reliable extrapolation method to this reaction, too.

The first purpose of this article is to theoretically predict the energy dependence of astrophysical S-factors for radiative charged particle reactions. The second purpose is to extrapolate the observed data to the low energy region (E_{cm} = 10 - 300 keV) which is inaccessible in the experiment, by making use of the inferred theoretical energy dependence. The third purpose is to compare the calculated result with experiments for several key reactions in order to assess the discrepancy of the energy dependence among different data as observed in the ^{4}He(^{3}H, γ)^{7}Li reaction.

§2. Theoretical Method

Theoretical estimate of the thermonuclear reaction rate requires an accurate wave function at the nuclear surface region. The microscopic cluster model[23] is one of the powerful tools for these phenomena and has in fact enjoyed a success for predicting the astrophysical S-factors for several cluster reactions like ^{4}He(^{3}He, γ)^{7}Be, ^{4}He(^{3}H, γ)^{7}Li [ref. 24], etc. However, it is not easy to extend the model to the other reactions in which the spectroscopic amplitude of the captured states is unknown experimentally. This is a common disadvantage in the potential model, too. In addition, the calculated absolute value of the reaction rate depends on the adopted effective interactions. Since the choice of the effective interaction is rather ambiguous, it seems best to use only the theortetical energy dependence in order to extrapolate the accurate high energy data to low energy.

A problem is how to predict the energy dependence of the S-factor in a sound way as independently of the specific nuclear models as possible. We here consider the kinematical conditions of the radiative charged particle reactions.[25,26]

2.1 Energy dependence of the scattering wave function

The incident scattering wave function is written as

$$\chi_{JL}(k, r) = 1/\sqrt{v}\,[F_L(k, r) + \tan\delta_J\, G_L(k, r)]/kr, \tag{1}$$

in the asymptotic region, where δ_J is the nuclear phase shift, and F_L and G_L are the regular and irregular Coulomb functions, respectively. Although the difference between the true wave function and eq. (1) is not ignored in the nuclear interaction region, the total radiative capture cross section is not affected very much by this difference when the incident energy satisfies the following relation

$$E_{cm} < E_0 = \mu/2(Z_1Z_2e^2/\hbar)^2, \tag{2}$$

and $E_{cm} << V_C = Z_1Z_2e^2/R_C$. It is useful to use two arguments $\eta\rho$ and η^2, where $\eta = Z_1Z_2e^2/\hbar v$ and $\rho = kr$, in order to consider the energy dependence of the Coulomb functions. The argument $\eta\rho$ is energy independent but η^2 is inversely dependent on energy, satisfying

$$\eta^2 = E_0/E_{cm}, \tag{3}$$

where E_0 is defined by eq. (2). The Coulomb functions are expanded at low energies in terms of the modified Bessel-Clifford functions I_m and K_m as[27]

$$F_L(k, r) = \sqrt{2\pi\eta/\{\exp(2\pi\eta)-1\}}\ \gamma_L(\eta^2)\ k(\eta^2)/2/(2\eta)^{L+1}\ (2\sqrt{2\eta\pi})$$

$$[I_{2L+1}(2\sqrt{2\eta\pi}) + \sum a_m(\eta^2)\ (-2\sqrt{2\eta\pi})^{m+1}\ I_{2L-m}(2\sqrt{2\eta\pi})], \tag{4}$$

$$G_L(k, r) = \sqrt{\{\exp(2\pi\eta)-1\}/2\pi\eta}\ 1/\gamma_L(\eta^2)/(2L+1)\ (2\eta)^L/(2L)!\ (2\sqrt{2\eta\pi})$$

$$[K_{2L+1}(2\sqrt{2\eta\pi}) + \sum a_m(\eta^2)\ (2\sqrt{2\eta\pi})^{m+1}\ K_{2L-m}(2\sqrt{2\eta\pi})], \tag{5}$$

Here, the functions $\gamma_L(\eta^2)$, $k(\eta^2)$ and $a_m(\eta^2)$ are defined by

$$\gamma_L(\eta^2) = 2^L/(2L+1)!\{(1 + \eta^2)(2^2 + \eta^2)...(L^2+\eta^2)\}^{1/2}, \tag{6}$$

$$k^{-1}(\eta^2) = 1/(2L+1)! + \sum 1/(2L-m)!\ (-2)^{m+1}a_m(\eta^2), \tag{7}$$

$$32\eta^2(m + 4)a_{m+3} = 2(2L - m - 2)a_{m+1} + a_m, \tag{8}$$

with $a_1(\eta^2) = L/16\eta^2$, $a_2(\eta^2) = 1/96\eta^2$ and $a_3(\eta^2) = L(L-1)/512\eta^4$. Although there are several different expansion methods of the Coulomb functions, the present form is the best to examine the convergence of many physical quantities.[26]

The nuclear phase shift δ_J is parametrized from the boundary conditions. In the direct captures eq. (1) is described by the scattering from a charged hard sphere of radius r_0, and hence the nuclear phase shift is written as

$$\tan\delta_J = - F_L(k, r_0)/G_L(k, r_0). \tag{9}$$

The value of r_0 is dependent on J and L and determined so that eq. (9) reproduces the obsereved phase shifts at higher energies (E_{cm} = 1 - 5 MeV).

Three point WKB method[28] is applied to δ_J when there is a sharp resonance near the threshold energy. The nuclear phase shift is given by

$$\tan\delta_J = (n-1)/(n+1)\ \tan[\pi/2\{1 + (E_{cm} - E_R)/\hbar\omega\}], \tag{10}$$

$$n - 1 = 1/2\ F_L(k, r_2)/G_L(k, r_2), \tag{11}$$

where E_R is the resonance energy, $h\omega$ is an energy quantum in the assumed harmonic oscillator well, and r_2 is the second turning point of WKB integral. Its power is demonstrated for several nuclear reactions.[28,29] The nuclear phase shifts (9) and (10) also are expanded in the similar way as eqs. (5) and (6).

Inserting eqs. (4) - (11) into eq. (1), we finally obtain the scattering wave function which is expanded in terms of $\eta^{-2} = E_{cm}/E_0$ as

$$\chi_{J=1/2,\,L=0}(k, r) = \sqrt{2\pi\eta/\{\exp(2\pi\eta) - 1\}}\ 1/\sqrt{v}\ \sum \chi^{(n)}(\eta\rho)\,(E_{cm}/E_0)^n, \tag{12}$$

where $\chi^{(n)}(\eta\rho)$ is a function that consists of the modified Bessel-Clifford functions and the polynomials of $2\sqrt{2\eta\rho}$.

2.2 Bound state wave function

We assume that the captured satates are weakly bound from the threshold of incident channel and approximated by

$$\chi_{JL}(r) = \vartheta_J \sqrt{3/r_N}\ W_{\eta' L}(r)/W_{\eta' L}(r_N)/r, \tag{13}$$

where $W_{\eta' L}(r)$ is the Whittacker function, $\eta' = Z_1Z_2e^2\mu/h^2\kappa$, $\kappa = \sqrt{2\mu BE}/h$, BE = binding energy, and ϑ_J is the reduced width (or equivalently the spectroscopic amplitude). r_N is the nuclear radius $1.2x(A_1^{1/3} + A_2^{1/3})$. Eq. (13) is smoothly connected to the harmonic oscillator wave function in the internal region. Note that the internal harmonic oscillator function does not contribute much to the capture cross section at astrophysical low energy.

Several different physical channels may affect the capture cross section. However, these contributions are negligible when the channels are closed at thermal energies and the tail of the admixed bound state wave function vanishes very fast with increasing r.

2.3 Energy dependence of astrophysical S-factor

The astrophysical S-factor for radiative capture reactions is defined by

$$S(E_{cm}) = E_{cm} \exp(2\pi\eta)\ \sigma(E_{cm}), \tag{14}$$

$$\sigma(E_{cm}) = \sum 8\pi(\lambda + 1)/\lambda\{(2\lambda + 1)!!\}^2\ 1/h\ (\omega/c)^{2\lambda+1}$$

$$\sum 1/(2S + 1)\ |<(L_fS_f)\,J_f\,||\,M^{E(M\,\lambda}\,||\,(L_iS_i)\,J_i>|^2, \tag{15}$$

where $\omega = (E_{cm} + BE)/h$ is the emitted photon energy and $M^{E(M)\lambda}$ is the electric (magnetic) λ-pole operator. There are two sources of the energy dependence of the S-factor. They are the trivial photon phase space factor $\omega^{2\lambda+1}$ and the energy dependence of the reduced matrix elements. The energy dependence $E_{cm} \exp(2\pi\eta)$ in eq. (14) countervalances the first two factors of eq. (12) squared, $2\pi\eta/\exp(2\pi\eta)/v$, when one takes an approximation $\exp(2\pi\eta) - 1 = \exp(2\pi\eta)$ which is valid at thermal energies. It is clear that the reduced matrix elements in eq. (15) are expanded by power series in terms of E_{cm}/E_0 by making use of eq. (12);

$$S(E_{cm}) = \sum g_\lambda (E_{cm}+BE)^{2\lambda+1} \sum h_{\lambda J_i L_i S}\ \vartheta_{Jf}^2\ |\int \chi_{Jf,Lf}(r) M_D{}^{E(M)\lambda}(r) \sum \chi^{(n)}(\eta\rho)(E_{cm}/E_0)^n\ r^2dr\,|^2, \tag{16}$$

$$= S(0)\ [\ 1 + a_1 E_{cm} + a_2 E_{cm}{}^2 + \dots\ ], \tag{17}$$

where g_λ and $h_{\lambda JLS}$ are the known factors. The coefficient ai in eq. (17) is calculated independently of the spectroscopic amplitude f of the final bound state, though S(0) is not. The expression of eq. (17) is convergent at the low energies satisfying eq. (2).

§3. Applications

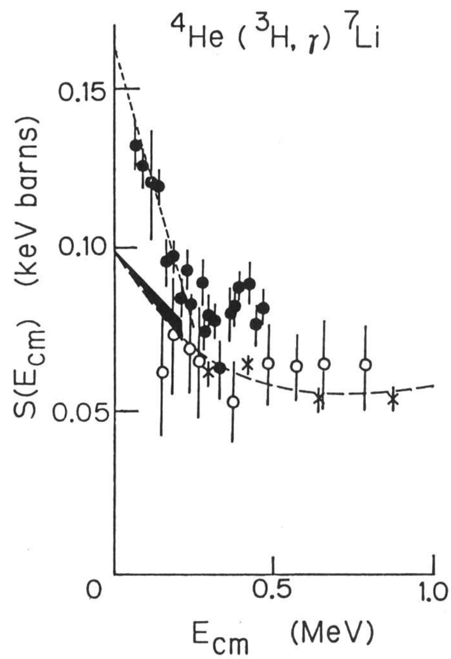

Fig. 1 S-factors for $^4He(^3H, \gamma)^7Li$. The open circles , crosses and closed circles are the observed data from refs. 7 - 9.

3.1 $^4He(^3H, \gamma)^7Li$: This reaction is dominated by the E1 transition with a very small admixture of the E2 transition at the 1% level and described by the direct capture process. The calculated energy dependence of the S-factor is shown in fig. 1 and compared with observations. In order to see the effects of the hard core radius r_0 on the energy variation of $S(E_{cm})$, r_0 is varied as $r_0 = 2.5 - 3.0$ fm, corresponding to the shaded area. S(0) is set equal to 0.098 keV barn in this figure which is a theoretical prediction[24] in the microscopic cluster model, though the normalization is arbitrary in the present theory. The obtained energy dependence for $r_0 = 2.8$ fm is given by

$$S(E_{cm})/S(0) = 1 - 1.15E_{cm} + 0.23E_{cm}^2,$$

which is in marginally agreement with the result calculated the microscopic theory (dashed curve in fig. 1). The Schroder et al. data[9] show much stronger energy dependence at the low energies as indicated by the dotted curve, but it is not justified theoretically.

3.2 $^7Be(p, \gamma)^8B$: The low energy ($E_{cm} < 500$ keV) reaction is dominated by the direct E1 process. Although there is a 1^+ resonance at $E_{cm} = 643$ keV, the contributing magnetic dipole transition to the ground 2^+ state does not have a long tail to the lower energy region as displayed in fig. 2. The incident s- and d-waves contribute to the E1 transition. We obtain

$$S(E_{cm})/S(0)_{s+d} = 1 - 1.68E_{cm} + 9.81E_{cm}^2,$$

using $r_0 = 4.1$ fm for both partial waves (solid curve). The signicance of d-wave contribution is observed clearly by switching off its contribution (dashed curve). The first logarithmic derivative of $S(E_{cm})$ becomes larger as $S(E_{cm})/S(0)_s = 1. - 2.22E_{cm} + 8.29E_{cm}^2$, which agrees with the potential model calculation done by Tombrello.[18]

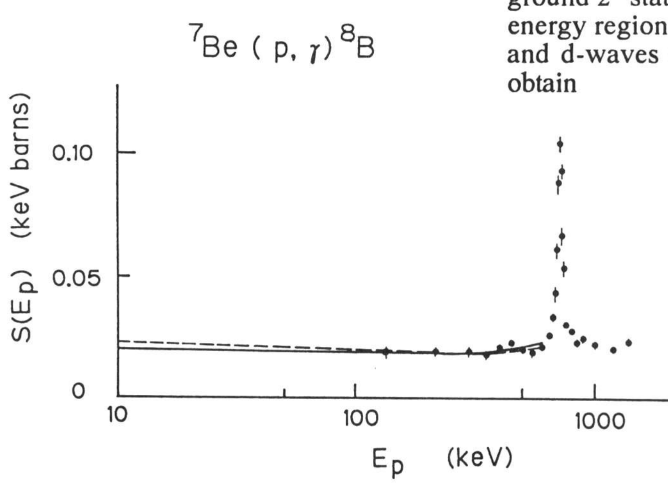

Fig. 2 S-factors for $^7Be(p, \gamma)^8B$. The dotts are the observed data from ref. 16.

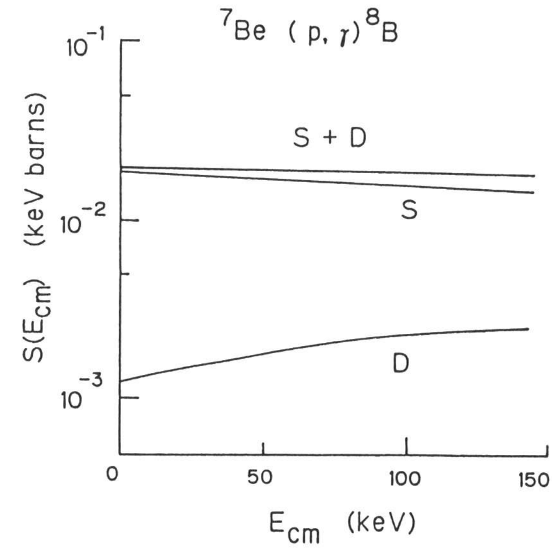

Fig. 3 s- and d-wave contributions to the S-factor for ^{7}Be(p, γ)^{8}B.

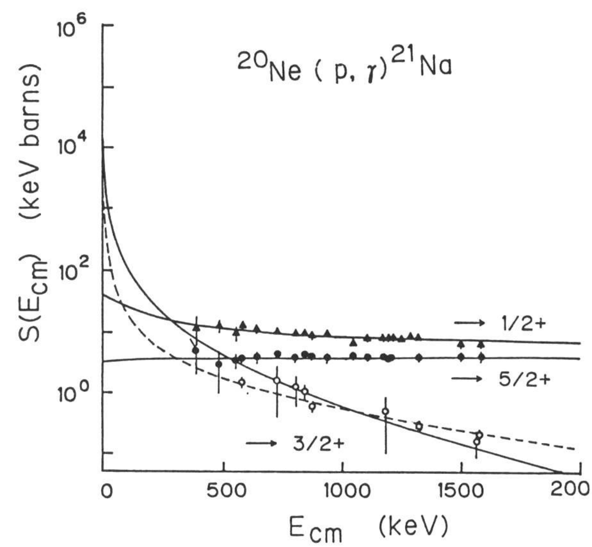

Fig. 4 S-factors for ^{20}Ne(p, γ)^{21}Na. The data points are from ref. 30.

(Power series expansion taking higher orders is used to give the two theoretical curves in fig. 2) Although the d-wave contribution is at the 10% level, its energy dependence is quite different from the s-wave as shown in fig. 3. In the present estimate the S(E_{cm} = 10 keV)-value is reduced by nearly 16% by taking account of the d-wave contribution, and the theoretical solar neutrino counting rate in the ^{37}Cl detector also is reduced by 13%.

A common core radius r_0 = 4.1 fm was assumed here, but the mirror nuclear system ^{7}Li(n, n)^{7}Li suggests a little different nuclear phase shifts from the hard core scatterings. More careful study is now going on.

3.3 ^{20}Ne(p, γ)^{21}Na: The low energy (E_{cm} < 150 keV) radiative transitions from the ^{20}Ne + p channel are dominated by the E2 transition to the ground ($3/2^+$) state and the E1 transitions to the first ($5/2^+$) and third ($1/2^+$) excited states. The E2 transition is affected by the so called high energy-tail of the $1/2^+$ state near the proton threshold, and eq. (10) is applied. The other two E1 transitions are the direct capture processes, and eq. (9) is applied.

The calculated results are shown in fig. 4. The astrophysical S-factors for the E 1 transitions ($5/2^+$ and $1/2^+$) are smoothly extrapolated to zero energy by making use of the theoretical energy dependence. The most striking is the strong energy variation of the E2 transition ($3/2^+$) rate, reflecting the high-energy-tail of the $1/2^+$ state. The extrapolated S-value of this tail is dominating the total reaction rate at low energies as concluded by Rolf[30] who has succeeded for the first time in observing the tail effects. The inferred energy dependence as displayed by solid curve, however, is different from the Breit-Wigner form (dashed curve). The present extrapolation results in S(E_{cm} = 100 keV) = 120 keV barn which is about 5 times larger than the value extrapolated by using the Breit-Wigner form.

3.4 ^{12}C(^{4}He, γ)^{16}O: Kremer et al.[31] has experimentally separeted the electric dipole contribution from the data. The high-energy-tail of the 42 keV bound 1^- state, however, is not clearly observed because of the large width of the second 1^- state. The present theory was applied to the first 1- state, and the result obtained agrees well with the energy dependence calculated in the microscopic clustr model.[32] The interference between the two 1^- states must be accounted for in order to extrapolate the observed data to the low energy E_{cm} = 300 keV which is the effective stellar temperature for helium burning.

§4. Conclusion

The kinematical conditions have been applied to formulate the energy dependence of astrophysical S-factors for charged particle reactions. There are many reactions that play an essential role in cosmic and stellar evolution. Some of these reaction rates are unknown or ambiguously measured at very low energies E_{cm} = 10 - 300 keV of astrophysical interest. The inferred theoretical energy dependence of the S-factor has been applied to extrapolate the observed data to this energy region. Although the present kinematical model of the radiative S-factor is simple, it works very well at the low energies provided that the final captured states are weakly bound, the low multipole radiations dominate the reactions and that the incident channel consists of the charged particles.

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