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The Baryon–Baryon Interaction in the SU(3) Skyrme Model

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Potentials among the octet and decuplet baryons are calculated under the product ansatz in the SU(3) Skyrme model. The potentials generally agree with conventional meson-exchange potentials at large distances. Some peculiality is, however, observed stemming from the commonly-used SU(3) quantization using rotational collective coordinates. The spin-isospin independent central potential is weakly attractive in intermediate range due to the strong chiral symmetry breaking term. The SU(3) model yields the isoscalar spin- dependent potential that does not appear in SU(2) models.

§ 1. Introduction

The Skyrme model,[1)] based on large N_c limits of quantum chromodynamics, has attracted much interest as a chiral soliton model of baryons. Under SU(2) treatments, the model has been phenomenologically successful in the general description of baryon static properties[2)] and of the nucleon-nucleon (NN) interaction.[3,4)] There are, however, some prominent phenomenological failures of the SU(2) model: the small nucleon axial coupling constant and the absence of central attraction in the NN interaction. The latter is crucial, so the model is not yet considered seriously as a useful model in nuclear physics.

The Skyrme model also has been extended to the SU(3) formulation for description of the static properties of the baryons, generally providing somewhat better agreement with the data than the SU(2) results.[5)] Some curious consequences, however, have been shown to emerge from the SU(3) model such as an appreciable strangeness content of the nucleon.[5,6and7)] It would be then interesting to examine baryon-baryon interactions in the SU(3) model. Our calculation reported here is the first of such examinations and is meant to be exploratory. In this spirit, we make the same assumption as that in many previous SU(3) calculations of baryon static properties,[5)] the SU(3) structure by embedding the SU(2) chiral fields, as shown in Eq.(2) below. As in the SU(2) formulation,[3,4)] we express the baryonic states by the collective rotational coordinate method and obtain adiabatic baryonic potentials under the product ansatz.

In the following, we describe briefly the SU(3) formulation of the Skyrme model and the baryon interactions in Sections 2 and 3, respectively. Details of our results are presented in Section 4, and the summary of our work is given in Section 5.

§ 2. The SU(3) Skyrme Model

The Skyrme Lagrangian is expressed as a sum of the kinetic energy term , ($\mathcal{L}_2$), the quartic term, ($\mathcal{L}_4$), the chiral symmetry breaking term ($\mathcal{L}_{\chi SB}$) and the Wess–Zumino–Witten term, ($L_{\rm WZW}$): [1)]

$$L = \int d^3r[\mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{\chi SB}] + L_{\rm WZW} \quad , \tag{1a}$$

$$\mathcal{L}_2 = -\frac{f_\pi^2}{16}\mathrm{Tr}[L_\mu L^\mu] \quad (1b), \quad \mathcal{L}_4 = \frac{1}{32e^2}\mathrm{Tr}\{[L_\mu, L_\nu]^2\}, \tag{1c}$$

$$\begin{aligned}\mathcal{L}_{\chi \rm SB} =& \frac{f_\pi^2(m_\pi^2 + m_\eta^2)}{32}\mathrm{Tr}\{U + U^\dagger - 2\} \\ &+ \frac{\sqrt{3}f_\pi^2(m_\pi^2 - m_K^2)}{24}\mathrm{Tr}\{\lambda_8(U + U^\dagger - 2)\},\end{aligned} \tag{1d}$$

$$L_{\rm WZW} = N_c\Gamma = \frac{iN_c}{240\pi^2}\int_Q d\Sigma^{\mu\nu\rho\sigma\alpha}\mathrm{Tr}\{L_\mu L_\nu L_\rho L_\sigma L_\alpha\}, \tag{1e}$$

where $L_\mu = U^\dagger \partial_\mu U$ denotes the left–handed current for an SU(3) matrix U, f_π is the pion decay constant, and e is a parameter related to the vector–meson coupling constant. m_π, m_η and m_K denote the masses of pion, η– and K– mesons, respectively.

The static chiral field U_0 is assumed to have the structure that the hedgehog SU(2) matrix $U_{SU(2)}(\vec{r}) = \exp(i\vec{\tau}\cdot\hat{r}F(r))$ is embedded in the SU(3) space:

$$U_0 = \begin{pmatrix} U_{SU(2)} & 0 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

The SU(3) baryonic states are then constructed from the rotational states $AU_0(\vec{r})A^\dagger$ with the SU(3) collective coordinates A. The collective coordinate A is parametrized as [6)]:

$$\begin{aligned}A =& \exp(-\frac{i}{2}\alpha\lambda_3)\exp(-\frac{i}{2}\beta\lambda_2)\exp(-\frac{i}{2}\gamma\lambda_3)\exp(-\frac{i}{2}\nu\lambda_4) \\ &\times \exp(-\frac{i}{2}\alpha'\lambda_3)\exp(-\frac{i}{2}\beta'\lambda_2)\exp(-\frac{i}{2}\gamma'\lambda_3),\end{aligned} \tag{3}$$

where, λ_a denotes the SU(3) Gell–Mann matrix. We note that the left multiplication of A by the SU(3) element generates a state in SU(3), while the right multiplication by SU(2) element induces the rotation in the configuration space. Thus, the left and right transformations of A are related to u–spin and spin transformations, respectively. The Euler angles α, β, γ and ν in Eq. (3) correspond to the Euler angles in the u–spin space, and α', β' and γ' those in the spin space. Hereafter, a, b and c are used as those of the SU(3) indices with values from 1 to 8; i, j and k the SU(2) indices with values from 1 to 3.

Following the canonical quantization rule for the Euler angles, we obtain the spin and u–spin differential operators as

$$A^\dagger\dot{A} = \frac{i}{2}\lambda_j J_j \quad \text{and} \quad -A\dot{A}^\dagger = \frac{i}{2}\lambda_a I_a, \tag{4}$$

respectively, in terms of the spin operator J and the u-spin operator I. J and I satisfy the commutation relations

$$
\begin{aligned}
&[J_i, J_j] = if_{ijk}J_k, \quad && [I_a, I_b] = if_{abc}I_c, \\
&[J_i, A] = A\frac{\lambda_i}{2}, && [I_a, A] = -\frac{\lambda_a}{2}A,
\end{aligned}
\tag{5}
$$

where f_{abc} are the SU(3) structure constants. The adjoint representation of A is defined as $D_{ai}(A) = \mathrm{Tr}\{\lambda_a A \lambda_i A^\dagger\}/2$. with the first index a corresponding to the u–spin and the second index j to the spin. This representation yields the fundamental representation of $SU(3) \times SU(2)$, equivalent to the SU(6) quark model.[5] For example, the unnormalized quark states with spin up are given by

$$
\begin{aligned}
\psi_{\mathrm{u},\uparrow} &= -(D(\Omega)_{++}D(\Omega')_{+-}\cos\nu + D(\Omega)_{+-}D(\Omega')_{--}), \\
\psi_{\mathrm{d},\uparrow} &= (D(\Omega)_{-+}D(\Omega')_{+-}\cos\nu + D(\Omega)_{--}D(\Omega')_{--}), \\
\psi_{\mathrm{s},\uparrow} &= -iD(\Omega')_{+-}\sin\nu,
\end{aligned}
\tag{6}
$$

where $D(\Omega)_{ij}$ and $D(\Omega')_{ij}$ denote the D–function of spin 1/2 with the Euler angles Ω $(\alpha\beta\gamma)$ and Ω' $(\alpha'\beta'\gamma')$, respectively. As the Skyrme model is the classical, the baryon states are expressed in terms of totally symmetric tensors in SU(6). The octet state has a [21] × [21] symmetry in $\mathrm{SU_{flavor}}(3) \times \mathrm{SU_{spin}}(2)$, and the decuplet state a [3] × [3] symmetry. For example, the proton state is represented as

$$
\begin{aligned}
\psi_{\mathrm{proton},\uparrow} &= -\cos\nu(D(\Omega)_{++}D(\Omega')_{+-}\cos\nu + D(\Omega)_{+-}D(\Omega')_{--}) \\
&= \cos\nu\, \psi_{\mathrm{u},\uparrow},
\end{aligned}
\tag{7}
$$

which is proportional to $\psi_{\mathrm{u},\uparrow}$,being in the state of 1/2 spin, 1/2 isospin, and 0 strangeness. The Euler angle ν does not appear in the SU(2) model and represents an SU(3) degree of freedom as shown in Eqs. (3), (6) and (7).

§ 3. The Baryon Interaction in the SU(3) Model

In order to extract the baryon-baryon potentials, we use the product ansatz for SU(3) matrices. The ansatz is known to be a poor variational ansatz particularly at short distances, but since no practical, alternative method is presently available, we have decided to use this conventional method. Our potentials will be therefore generally of semiqualitative significance but are expected to be reasonably reliable in the asymptotic regions. This procedure has the advantage that the potentials may be compared with our SU(2) results previously carried out in a similar way.[4] Under the product ansatz, we have

$$
U(\vec{r}, A_1, A_2, \vec{R}) = U_2(\vec{r} - \frac{\vec{R}}{2}, A_2)U_1(\vec{r} + \frac{\vec{R}}{2}, A_1) \tag{8a}
$$

$$
U_1(\vec{r} + \frac{\vec{R}}{2}, A_1) = A_1 U_0(\vec{r} + \frac{\vec{R}}{2})A_1^\dagger, \quad U_2(\vec{r} - \frac{\vec{R}}{2}, A_2) = A_2 U_0(\vec{r} - \frac{\vec{R}}{2})A_2^\dagger, \tag{8b}
$$

where A_1 and A_2 are rotational collective coordinates.

We obtain the interaction Hamiltonian by substituting Eq.(8) into Eq.(1) and classify various interaction terms according to the spin-isospin degree of freedom. Classification is done using the adjoint representation, $D_{ij}(A_1^\dagger A_2) = \mathrm{Tr}\{\tau_i A_1^\dagger A_2 \tau_j A_2^\dagger A_1\}/2$ of the combined collective coordinates as $A_1^\dagger A_2$, where the indices i and j denote the spins for A_1 and for A_2, respectively. In our formalism, the SU(2) soliton is embedded in the SU(3) as in Eq.(2), and the domain of D_{ij} is thus effectively restricted to SU(2). Accordingly, while the actual values of the matrix elements differ from those in the pure SU(2) case, the structure of the interaction terms is formally similar to the SU(2) structure. We refer the reader to our previous SU(2) work[4)] for a detailed description of the formalism presented in this section.

The interaction terms are expressed in terms of integrals of the left and right currents that consist of the chiral fields and the D_{ij}'s. The expressions are simplified using the tensor decomposition

$$D_{ij}(A)D_{kl}(A) = \frac{1}{3}\delta_{ik}\delta_{jl} - \frac{1}{6}\delta_{ij}D_{kl}(A) - \frac{1}{6}\delta_{kl}D_{ij}(A) + \frac{1}{6}\delta_{il}D_{kj}(A) + \frac{1}{6}\delta_{kj}D_{il}(A) + \text{Higher order tensor terms}, \tag{9}$$

by exploiting the fact that the presentation is the second order. Note that $D_{ij}(A_1^\dagger A_2) = Dai(A_1)Daj(A_2)$, because $D_{ij}(A_1^\dagger A_2)$ is a representation of SU(3). Another major technique used for simplification concerns the left current L_k;

$$\begin{aligned} L_k &= A_1^\dagger A_2 L_k(U_0(\vec{r} - \frac{\vec{R}}{2}))A_1^\dagger A_2 - R_k(U_0(\vec{r} + \frac{\vec{R}}{2})) \\ &= i[A_1^\dagger A_2 \tau_i A_2^\dagger A_1 L_k^{i(2)} - \tau_i R_k^{i(1)}] \\ &= i\lambda_a[D_{ai}(A_1^\dagger A_2)L_k^{i(2)} - \delta_{ai}R_k^{i(1)}]. \end{aligned} \tag{10}$$

Here, the domain of $D_{ai}(A_1^\dagger A_2)$ which is restricted to the SU(2) or SU(3) indices are actually restricted to the SU(2) indices, because the adjoint representation D_{ab} is truncated with the SU(2) values of the left and right currents. Accordingly, the first and second indices of $D_{ai}(A_1^\dagger A_2)$ again correspond to the spin of the A_1 and A_2 skyrmions, respectively. The Wess–Zumino–Witten term $L_{\rm WZW}$ does not contribute to the adiabatic potential because the terms in $L_{\rm WZW}$ includes time derivative even after integration on 5–dimensional disk.

Using Eq. (3), we finally obtain the expressions of the interactions as

$$V_2 = \frac{f_\pi^2}{4}\int d^3r(\tilde{R}L) \tag{11a}$$

$$V_4 = \frac{1}{2e^2}\int d^3r[2(RRR\tilde{L}) + 2(LLL\tilde{R}) + (RRLL) + (\tilde{R}L\tilde{R}L) + (\tilde{R}LL\tilde{R})] \tag{11b}$$

$$\begin{aligned} V_{\chi\rm SB} = -\frac{f_\pi^2}{8}\int d^3r[&(m_\pi^2 + m_\eta^2)(U_+ - 1)_0(U_- - 1)_0 \\ &+ (U_+ - 1)_j(U_- - 1)_k\{(2m_\pi^2 + m_\eta^2 - m_K^2)\sum_{l=1}^{3} D_{lj}(A_2)D_{lk}(A_1) \\ &+ (\frac{1}{2}m_\pi^2 + m_\eta^2 + \frac{1}{2}m_K^2)\sum_{l=4}^{7} D_{lj}(A_2)D_{lk}(A_1) \\ &+ (m_\eta^2 + m_K^2)D_{8j}(A_2)D_{8k}(A_1)\}] \end{aligned} \tag{11c}$$

where the subscripts of V identify the corresponding lagrangian terms in Eq. (1). Furthermore, (AB) and $(ABCD)$ stand for $A_j^iB_j^i$ and $A_j^iB_j^iC_k^lD_k^l - A_j^iB_k^iC_j^lD_k^l$, [where $\tilde{R}_k^j$ $(\tilde{L}_k^j)$ denotes $R_k^{i(1)}D_{ij}(A_1^\dagger A_2)$ $(D_{ij}(A_1^\dagger A_2)L_k^{i(2)})$], respectively; $(U_\pm - 1)_j$ denotes

$\mathrm{Tr}\{\tau_j(U_\pm - 1)/2\}$ for $j = 0, 1, 2$ and 3.

The potential between specific baryons is obtained by taking matrix elements of $D_{ia}(A)$. For example, the nonzero matrix elements between the nucleons are tabulated in Table I.

Table I. Nonzero matrix elements of the adjoint representation D_{aj} for spin up nucleon states.

D_{13}	$-iD_{23}$	D_{33}		$\sqrt{3}D_{83}$	
$<p,p>$	$<p,n>$	$<p,p>$	$<n,n>$	$<p,p>$	$<n,n>$
-7/30	7/30	-7/30	7/30	-1/10	-1/10

§ 4. Numerical Results and Discussion

Equation (11) shows some of characteristic features of the baryon interactions in the SU(3) model:

(i) the radial form of each potential term is independent on the baryon configuration.

(ii) the spin dependent terms have numerical factors different from the SU(2) results. The difference stems from the enlargement of the spin space to the u-spin space, and the matrix elements of D_{aj} for baryon states are modified. For example, the nucleon amplitude in the baryon multiplet is reduced following the reduction of the the nucleon matrix element $D_{aj}(A)$ from $-1/3\tau_i\sigma_j$ [SU(2)] to $-7/30\tau_i\sigma_j$ [SU(3)]. The nuclear spin–spin and tensor potentials contributed by $\mathcal{L}_2$ and $\mathcal{L}_4$ are thus reduced by 51 % in magnitude.

(iii) The isoscalar spin–spin and tensor terms emerge that do not appear in the pure SU(2) model. The new terms are generated because $D_{aj}(A)$'s for $a \geq 4$ no longer vanish. The isoscalar terms appear in all baryon interactions. Paticularly, the general NN potential form in the SU(3) model is

$$V = V_c + (V_{\mathrm{ss}}^0 + V_{\mathrm{ss}}^1 \tau \cdot \tau)(\sigma \cdot \sigma) + (V_{\mathrm{T}}^0 + V_{\mathrm{T}}^1 \tau \cdot \tau) S_{12}. \tag{12}$$

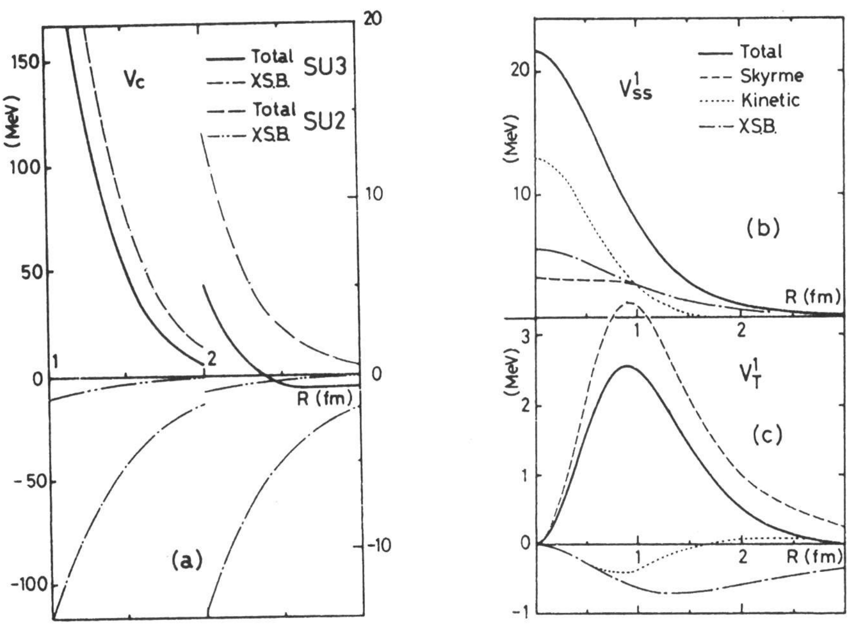
Note that, when the chiral symmetry breaking term is neglected, $V_{\mathrm{ss}}^0/V_{\mathrm{ss}}^1 = V_{\mathrm{T}}^0/V_{\mathrm{T}}^1 = 3/49$ for the NN potential from Table I.

(iv) The symmetry breaking term contributes appreciably, reflecting the fact that the SU(3) symmetry is not well conserved. For example, the first term in Eq. (11c) is increased by a factor of about 8 by the inclusion of m_η, compared with the SU(2) case. The increase enhances the attraction of the central potential. On the other hand, for example, the isovector spin-dependent NN potential changes little compared to the pure SU(2) model, reflecting the fact thast the SU(2) symmetry remains well conserved.

The effective potential for each BB channel can be obtained by diagonalizing of the potential matrix. In the present analysis, the chiral symmetry breaking term is treated perturbatively. Figure 1 illustrate some features of the NN and $N\Lambda$ potentials in the SU(3) model. In this calculation, we adopt the parameter set refered to Case III in ref.4. That is, the experimental data are used for the meson masses (m_π, m_K and m_η), and for the pion decay constant f_π. The parameter e is taken as 3.4.

Figures show the following characteristics on the NN potential:

(i) In the spin–isospin independent potential (Fig.1(a)), the χSB term is strongly attractive and has a range longer than that of the Skyrme term. Consequently, the resultant potential is only weakly attractive in the region beyond $r = 2$ fm, not sufficient to reproduce the observed attraction.



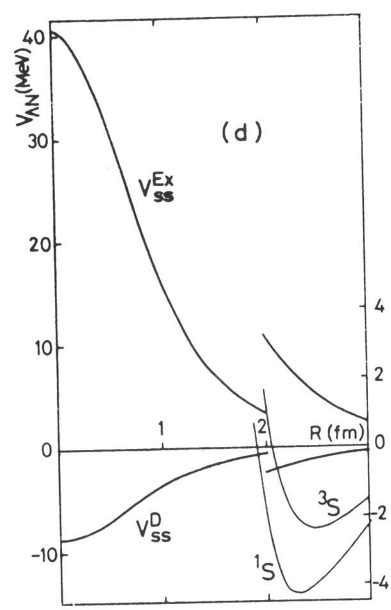


Fig. 1. The calculated NN– and ΛN– potentials. (a) is the spin-isospin independent central potential in both SU(2) and SU(3) models. (b) and (c) are the spin-spin ($V^1_{\rm ss}$) and tensor parts ($V^1_{\rm T}$) for the NN-potential, respectively. (d) is for the singlet and triplet s-state (^1S and ^3S) ΛN– potentials, and the spin-spin direct and exchange potentials ($V^{\rm D}_{\rm ss}$ and $V^{\rm Ex}_{\rm ss}$).

(ii) In the spin–isospin dependent potential((b) and (c)), the strength is reduced compared with the SU(2) case, following the reduction of the matrix element $< N \mid D \mid N >$ by 51 %.

(iii) In the outer region ($r \gtrsim 2.0$ fm), the potential is consistent with the Paris potential.

The ΛN potential is divided into the direct $V^{\rm D}$ and exchange $V^{\rm Ex}$ parts, corresponding to η- and K-meson exchange, respectively. Each part further consists of three terms: spin-independent central, spin-spin, and tensor terms. In Fig. 1d), we illustrate the ΛN potential of the triplet and singlet s-states, as well as the spin-spin dependent potentials ($V^{\rm D}_{\rm ss}$ and $V^{\rm Ex}_{\rm ss}$) in the direct and exchange parts as a representative case. Our ΛN potential agrees rather well with the conventinal meson exchange SU(3) potential,[8] including the relative strengths and signs of various terms. An exception is the asymptotic behavior of the potential that is governed by the pion mass instead of the η-meson and kaon masses. This peculiarity is a consequence of the simple SU(3) strucrure of the static matrix U_0 assumed from the outset, as shown in Eq. (2). Our calculation explicitly shows a shortcoming of this relative popular assumption. The otherwise good agreement with the meson-exchange potential indicates, however, that the use of U_0 structure more agreeable to SU(3) symmetry breaking would remove the peculiality.

§ 5. Concluding Remarks

The adiabatic baryon potentials are obtained in the SU(3) Skyrme model under the product ansatz using the rotational collective coordinates. The potential in all channels are found to be generally consistent with the conventinal meson-exchange potentials, including newly derived isoscalar spin-dependent terms. This is our major result.

When we compare the potentials with our previous SU(2) calculation carried out using the same method, we find that the most significant feature of the SU(3) model is the large contribution from the chiral symmetry breaking term. The term provides a weak NN attraction, giving hope that inclusion of distortion effects[9] may increase the attraction so that it compares more favorably with the observed strength.

The large contribution of the chiral symmetry breaking term raises, however, the question on the adequacy of our quantization procedure that implicitly assumes that there is no symmetry breaking. Furthermore, the peculiar asymptotic radial dependence in our ΛN potential explicitly shows that the commonly-used, simple SU(3) structure of Eq. (2) is inadequate. A recent proposal, the bound strange-meson approximation,[10] carries out the SU(3) quantization explicitly breaking the SU(3) symmetry from the outset. Perhaps our calculation should be repeated to examine effects of such a quantization procedure.

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