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Quantum Phase Shift of de Broglie Waves by Spatial Confinement

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A de Broglie wave will suffer a small change of momentum in the direction of motion following the introduction of transverse spatial constrictions. This results in a change in phase of the wavefunction that is purely geometrical in origin. This predicted phase change will be measured by introducing the constrictions into a perfect crystal neutron interferometer at the University of Missouri research Reactor Center.

KEYWORDS: de Broglie wave, quantum phase shift, spatial confinement

§.1. Introduction

In 1987 Levy-Leblond,[1] and independently Greenberger,[2] devised an experiment to observe the change in phase of thermal neutrons caused by spatial boundry conditions.

We are carrying out this experiment at the University of Missouri Research Reactor Center using a skew-symmetric perfect silicon crystal neutron interferometer and an array of square capillary tubes in order to test this proposition.

§.2. Quantum mechanical foundations

Consider a thermal neutron travelling in the z-direction with a well defined momentum p, mass m, incident on a channel of square cross-section of side length a, length ℓ, and possessing impenetrable walls. Assume that $\lambda \langle\langle a \langle\langle \ell$.

The incident asymptotic wave function of the thermal neutron may be expressed as

$$\Psi_{inc} \propto e^{ipz/\hbar} \tag{1}$$

with energy,

$$E = p^2/2m \tag{2}$$

and de Broglie wavelength,

$$\lambda = h/p \tag{3}$$

Inside the constriction, the boundary conditions,

$$\Psi(x=0) = \Psi(x=a) = \Psi(y=0) = \Psi(y=a) = 0 \tag{4}$$

give rise to a solution of the time-independent Schrödinger Equation,

$$\Psi_{n_x,n_y} \propto \sin(n_x \pi x/a)\sin(n_y \pi y/a)e^{ip'z/\hbar}, \tag{5}$$

where n_x and n_y are positive integers. This has an energy,

$$E = p^2/2m + (n_x^2 + n_y^2)\pi^2\hbar^2 / 2ma^2 . \tag{6}$$

That is, the momentum of the neutron has been changed by an amount Δp with the introduction of constrictions.

For the simplest case where $n_x = n_y = 1$, (higher modes will be below detectable limits)

$$p = \sqrt{2mE} \text{ and } p' = p\sqrt{1 - \pi^2\hbar^2/mEa^2} . \tag{7}$$

Assuming $\Delta p \langle\langle p$ and using the relation,

$$\Delta\Phi \approx \ell\Delta p / \hbar \tag{8}$$

we obtain the change of phase,

$$\Delta\Phi \cong (\pi/2)(\lambda\ell/a^2) . \tag{9}$$

Similarly for the case where the constrictions are one dimensional, i.e., a transmission grating, the effect is a factor of two smaller.

§.3. Expected Results

A graph of the expected change in phase of 2.35Å neutrons as a function of constriction side length, a, is shown in Fig. 1.

For example, the change of phase of a beam of neutrons with a wavelength of 2.35Å incident on a square cross-section constriction of side length 20 microns and length 30mm is expected to be 0.11 radians.

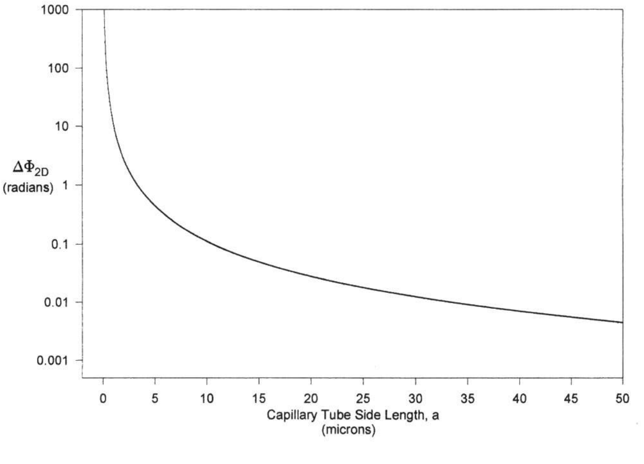

Fig. 1. Graph of expected change of phase as a function of constriction side length, a.

§.4. Experiment

4.1. Neutron Interferometer

The phase change will be measured using a perfect silicon crystal skew-symmetric four blade neutron interferometer shown in plane view in Fig. 2. The interferometer operates by a series of four Laue transmissions from the (220) crystal planes with the incident thermal neutrons satisfying the Bragg condition.[3) The four blades produce mirror and beam splitting effects in a topology analogous to the Mach-Zehnder optical interferometer.

The aluminium phase plate is rotated to produce a variable phase shift and sets the operating point on the interferogram. Masked constrictions will then be placed in one path of the beam, and the phase shift is directly measured.

4.2. Nature of the constrictions

The constrictions are composed of an array of square channel borosilicate capillaries constructed by a patented process[4) developed at the University of Melbourne.

A photograph of the cross-section of such an array with an open area greater than 70% and an aspect ratio, array length to channel side length, of 1000, as shown in Fig.3.

The geometry and surface quality of the interiors of the capillaries is sufficiently good to give high reflectivity at angles below the critical angle, as demonstrated by X-ray experiments.

4.3. Progress Report

An optical analogue of the proposed experiment has been performed and verified the principle. Calibration and characterization of the square channel capillary arrays of a variety of sizes is currently underway with soft X-rays. Preliminary experiments with neutrons have also commenced and are expected to conclude in 1996.

§.5. Conclusion

In this experiment, the phase of the neutron de Broglie wave is changed by virtue of spatial confinement alone, i.e., a longitudinal momentum change occurs with no applied force. In that sense it is analogous to Aharonov-Bohm effects and demonstrates another hitherto unverified non-local aspect of quantum mechanics.

1)J.-M. Levy-Lebond, *Phys. Lett.* A 125, 441 (1987)
2)D.M. Greenberger, *Physica* B 151, 374 (1988)
3)S. A. Werner and A. G. Klein in '*Methods of Experimental Physics*', Ed. B. Skjold, Vol23A, Academic Press NY (1986)
4)Provisional Australian Patent number PN3860 (1995)

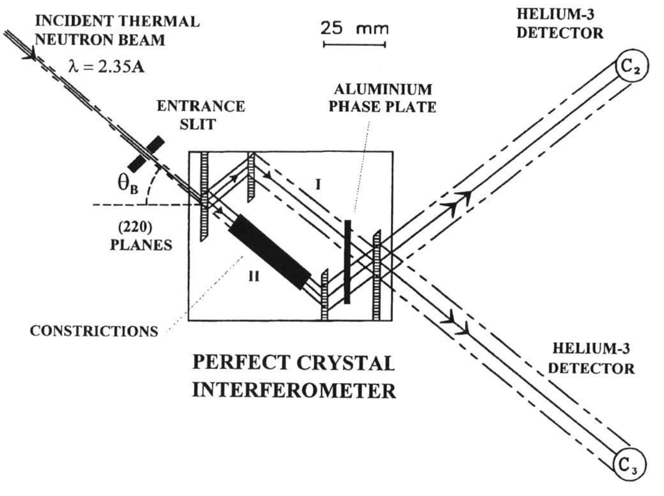

Fig. 2. Schematic of the expected experimental arrangement showing the constrictions inside the crystal interferometer.

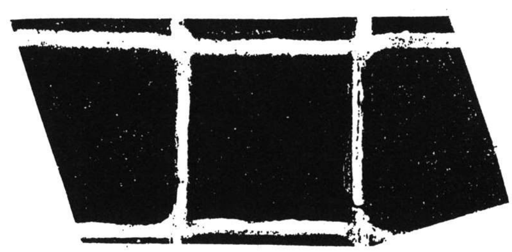

Fig. 3. Photograph of the cross-section of a typical square channel constriction.