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Geometric Phase in a Split-Beam Experiment Measured with Coupled Neutron Interference Loops

Yuji HASEGAWA, Michael ZAWISKY, Helmut RAUCH, and *Alexander IOFFE

Atominstitut der Österreichischen Universitäten, Schüttelstraße 115, A-1020 Wien, Austria
**Berlin Neutron Scattering Center, Hahn-Meitner Institut, Glienickerstraße 100, D-14109 Berlin, Germany*

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A geometric phase factor is derived for a split-beam experiment as an example of cyclic evolutions. The geometric phase is given by one half of the solid angle independent of the spin of the beam. We observe this geometric phase with a two-loop neutron interferometer, where a reference beam can be added to the beam from one interference loop. All the experimental results show complete agreement with our theoretical treatment.

KEYWORDS: neutron interferometer, geometric phase, Poincaré sphere

1. Introduction

During the last decade, the geometric effect on the phase of the wave function has excited considerable interest. It was Berry[1] who first clearly described the geometric phase factor for a quantum system transported adiabatically through a curve C in parameter space, this phase factor depending solely upon the geometry of the curve C. Several experiments were reported to manifest the effect of this geometric phase. A spinning light experiment in an optical fiber[2] was the first of this kind. A similar experiment was accomplished with a neutron beam in an adiabatically rotating magnetic field.[3,4] Aharanov and Anandan[5] released the restriction of adiabaticity for Berry's phase so that the geometric phase may be generalized to the *state* of the system in a cyclic evolution, i.e., that it returns to its initial state after an evolution. Experiments to observe this Aharanov-Anandan (AA) geometric phase were accomplished by laser interferometry[6,7] and nuclear magnetic resonance.[8] In addition, a dynamical aspect of the evolving geometric phase[9] and the geometric effect for noncyclic evolution[10] were demonstrated experimentally.

More recently, another example was shown of geometric phases in cyclic excursion around a diabolic point.[11] The experimental accomplishment to show the geometric property independent of the spin or polarization of the beam remains a minority compared to the spin or polarization associated work. In a split-beam experiment[12] where an incident beam is split and recombined, one can insert phase shifters and/or absorbers into each split beam, so that the system evolves under the action of two separate Hamiltonians. Here, we justify the split-beam experiment as a cyclic evolution of a quantum system by analogy to the spinor rotation and show a geometric property in it. In our theoretical treatment, the *overall* phase is considered as a sum of weighted phases of the two superposed partial beams, and the dynamical and the geometrical phase factors are derived for the cyclic evolution. Its geometrical property is shown with the use of Poincaré sphere descriptions. The four-plate neutron interferometer with two interference loops enabled us to realize experiments to observe this geometric phase, and the experimental results completely agree with the theoretical treatment.

2. Principle of the Experiment

It is well-known that the spinor rotation of a spin-$\frac{1}{2}$ particle in a homogeneous magnetic field results in a geometric phase.[1,5] Two bases, spin-up and spin-down states, are assumed in such a case. In the split-beam experiment, two similar bases are observable. This similarity justifies regarding neutron interferometry, which is an example of the split-beam experiment, as a cyclic evolution of the quantum system. In the split-beam experiment, the phase shifter, which is inserted to observe the interference oscillations, directs the evolution of the system along a certain curve, C, and the absorber, which reduces the intensity of one beam, changes this curve of the evolution.

The similarities between the spinor rotation and the split-beam experiment allow us to define the dynamical phase for the split-beam experiment. The dynamical phase, Φ'_D, can be defined as

$$\Phi'_D = \frac{\int_l \langle\Psi|\Delta \boldsymbol{k}\cdot d\boldsymbol{l}|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \frac{\int_{l_\mathrm{I}} \langle\Psi_\mathrm{I}|\Psi_\mathrm{I}\rangle \Delta \boldsymbol{k}_\mathrm{I}\cdot d\boldsymbol{l}}{\langle\Psi|\Psi\rangle} + \frac{\int_{l_\mathrm{II}} \langle\Psi_\mathrm{II}|\Psi_\mathrm{II}\rangle \Delta \boldsymbol{k}_\mathrm{II}\cdot d\boldsymbol{l}}{\langle\Psi|\Psi\rangle} \quad (1)$$

where $|\Psi_i\rangle$, $\boldsymbol{l}_i$, and $\boldsymbol{k}_i$ represent the two wavefunctions, beam paths, and wavevectors in the interferometer, respectively. Here, we omit the real part of the phase shift due to the absorber, which does not reduce the general validity of our treatment.

Rearranging Eq.(1), we get

$$\Phi'_D = \frac{I_\mathrm{I}}{(I_\mathrm{I}+I_\mathrm{II})}\times \chi'_\mathrm{I} + \frac{I_\mathrm{II}}{(I_\mathrm{I}+I_\mathrm{II})}\times \chi'_\mathrm{II} = \left(\frac{1}{1+T}\right)\times(\chi'_\mathrm{I}+T\chi'_\mathrm{II}) \quad (2)$$

* Permanent Address: St. Petersburg Nuclear Physics Institute, Gatchina, 188350, Russia

where I_i and χ'_i, respectively, represent intensities and phase shifts of the two beams, and T is the transmission probability of the absorber in the beam path-II.

The geometric phase, β', in the split-beam experiment is given with the total phase shift, ϕ', as

$$\beta' = \phi' - \Phi'_D \, . \tag{3}$$

From Eq.(2), one can see that, when rotating the phase shifter, the dynamical phase shift, $\Delta\Phi_D$, during the cyclic evolution becomes zero when

$$\Delta\chi'_{\mathrm{I}} + T \cdot \Delta\chi'_{\mathrm{II}} = 0 \, , \tag{4}$$

where $\Delta\chi'_i$ is the change of χ'_i during the cyclic evolution. This equation shows that β' is explicitly observable in the split-beam experiment with the right combination of phase shifters and absorbers.

It is very instructive to use the Poincaré sphere for the split-beam experiment and thereby to recognize the geometric nature of the derived geometric phase, just as the spin-sphere was used for the spinor evolution. The split-beam experiment can be completely described within the framework of a two-dimensional Hilbert space, H_2, where the states can be visualized as elements of the Poincaré sphere.[14] This sphere is shown in Fig.1. The vertical axis represents the relative intensity of the two beams, and the polar points represent the single-beam situations. When shifting the relative phase, θ, between the two beams, the state traces a latitudinal circle on the sphere dependent on the ratio between the intensities of the two beams.

With this sphere, the solid angle, Ω, which is subtended by the traced curve at the origin, is given by

$$\Omega = 2\pi \cdot \left(1 - \frac{1-T}{1+T}\right) = 4\pi \frac{T}{1+T} \, . \tag{5}$$

The geometric phase, $\beta'(C)$, for one cycle curve, C, is associated with its solid angle, $\Omega(C)$. Berry[1] has shown that this is given with the helicity, σ, by

$$\beta'(C) = \sigma\Omega(C) = 4\pi\sigma \cdot \frac{T}{1+T} \, . \tag{6}$$

Equation (5) shows that the solid angle, Ω, depends only on the transmission probability, T, of the absorber, which determines the curve of the evolution, and the geometric phase, $\beta'(C)$, is derived from this transmission probability.

When one uses the right combination of phase shifters and absorbers so that the condition of Eq.(4) is satisfied, the geometric phase emerges explicitly in the out-going beam from one interference loop. Since it is necessary to add a reference beam to the beam, which is recombined from one interference loop, a four-plate neutron interferometer with two loops is the most suitable tool for the detection of the geometric phase[13]; the experimental setup is shown in Fig.2. In the interference loop (Loop-A) between the second and the fourth plate of the interferometer, appropriate pairs of phase shifters and absorbers are inserted in each split beam path to compensate for the dynamical phase during the cyclic evolution. The phase shifters direct the state of the recombined beam from this

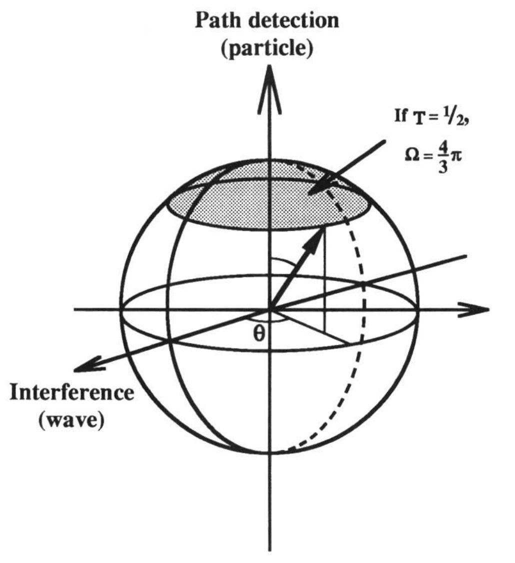

Fig.1 Poincaré sphere descriptions for split-beam experiments. Depicts the condition with unequal beam intensities, i.e., with an absorber.

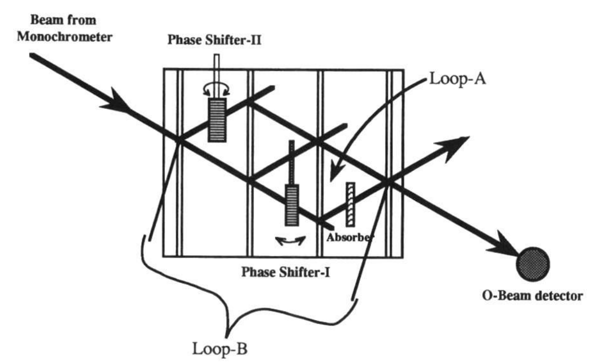

Fig.2 Experimental setup to measure the geometric phase in the split-beam experiment with a four-plate neutron interferometer.

interference loop through cyclic evolutions, and the absorber changes the curves of these cyclic evolutions. In the other interference loop (Loop-B), a beam split at the first plate of the interferometer is recombined with the beam from the interference Loop-A. This split beam acts as a reference, and another phase shifter is inserted in this beam path. The interference oscillations between the reference and the interference beams are measured using this additional phase shifter. The geometric phase of the out-going beam from the interference Loop-A is measured as shifts of these interference oscillations.

3. Experimental

The experiments were performed with the neutron interferometer instruments V9 at the BENSC, Hahn-Meitner Institut in Berlin.[15] A schematic view of the whole experimental arrangement is shown in Fig.2. A four-plate neutron interferometer of monolithic perfect silicon crystal having two interference loops was used.[13] One interference loop (Loop-A) between the second and the fourth plate of the interferometer is used to cause the evolution of the geometric phase. The other loop (Loop-B), which has an additional phase shifter, is used to observe the shifts of oscillations due to the geometric phase. This interferometer was adjusted to give (220) reflections. The 220-planes were perpendicular to the plates' surfaces. The wavelength was 1.95Å. The beam cross-section was reduced to 2mm (horizontal) and 10mm (vertical) by a Cd diaphragm in front of the interferometer. As shown in Fig.2, a ^{3}He-detector was set at one of the beams, this being in the transmittted direction after having been recombined from three beams.

In the interference Loop-A, an absorber and a phase shifter—which had different thicknesses for the two beams—were inserted. The absorber reduced the intensity of one of the beams, thereby changing the geometry of the evolution of the state. The relative phase between the two beams was changed by the phase shifter to promote the evolution of the state on a latitudinal circle on the sphere. At first, no absorber was inserted in order to examine the case when the intensities of two beams were the same. Later we inserted two different kinds of absorbers in one of the beam paths between the third and the fourth plate of the interferometer.

A phase shifter, called the phase shifter-I (PS-I), was inserted between the second and the third plate of the interferometer. Three kinds of parallel-sided Al plates were used, these having different thicknesses according to the transmission probability of the absorbers, so that the dynamical phase shifts during the evolution were zero. The first of these plates was 5mm in thickness for both split beams and was used for the case without the absorber. The second was 10mm thick for the beam which had been reduced in intensity by the absorber and 5mm thick for the other beam, and was used for the case in which the transmission probability of the absorber was 0.49. The third was 10mm thick for the intensity-reduced beam and 2mm thick for the other, and was used for the case in which the absorber had a transmission probability of 0.21. Rotation of these Al plates around the vertical axis produced a phase shift $\Delta\chi_i = -N\lambda b_c \Delta D_i$ on each beam, where N is the number of nuclei per volume, λ is the wavelength of neutrons, b_c is the coherent scattering length of Al, and ΔD_i is the change in thickness when Al plate is rotated.

The beam in the interference Loop-B, which was split at the first plate of the interferometer and recombined with the beams from the other interference loop (Loop-A) at the fourth plate, is used as a reference for the phase. In order to obtain an adjustable phase reference, a parallel-sided Al plate which was 5mm thick was inserted in the reference beam path between the first and the second plate of the interferometer. We call this plate the phase shifter-II (PS-II). Rotation of this Al plate around the horizontal axis changed the effective thickness of this plate in the beam and thus introduced the phase shift.

Before measuring the shifts of the interference oscillations with the phase shifter-II, it was necessary to show with the phase shifter-I how the interference loop-A would behave with various pairs of phase shifters and absorbers. Interference oscillations with three different pairs of phase shifters-I and absorbers were measured by rotating the phase shifter-I.

The geometric phase shifts, which we intended to measure, were induced on the recombined beam from the

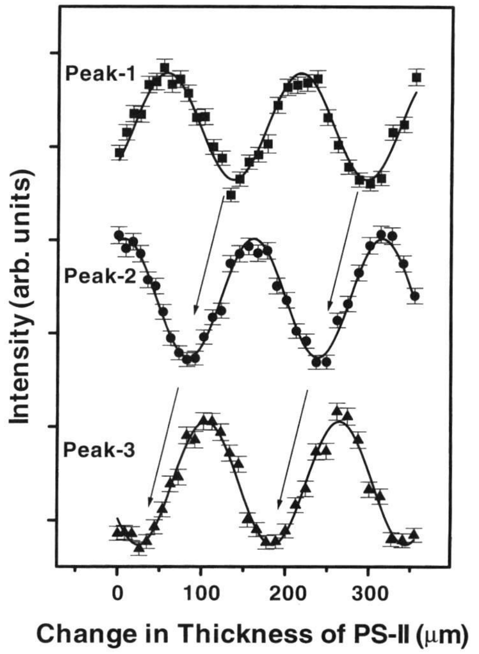

Fig.3 Typical shifts of interference oscillations measured with the Phase Shifter-II at the three peak positions.

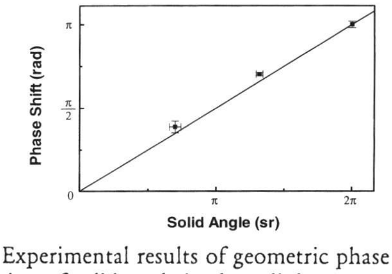

Fig.4 Experimental results of geometric phase shift as a function of solid angle in the split-beam experiment. The solid line corresponds to the theoretical prediction given by Eq.(6) for $\sigma = \frac{1}{2}$.

interference loop-A with the phase shifter-I and the absorber. These geometric phase shifts were measured with the phase shifter-II in the reference beam. Here, since we pay particular attention to the phase shift by the cyclic evolution of the system, interference oscillations caused by the PS-II were collected by fixing the PS-I at the three peak positions of the intensity modulations. The collected data were fitted to sinusoidal curves by the least-squares method. Typical intensity oscillations obtained by the PS-II, along with fitting curves and their shifts in peak position, are shown in Fig.3. One can see that the oscillations get shifted depending on the peak positions by the PS-I.

We collected the data by repeating the same measurements for the three combinations of the PS-I and the absorber. The obtained intensity modulations were fitted to sinusoidal curves and the shifts of the oscillations were analyzed quantitatively. With these procedures, we obtained the shifts of 3.144(38), 2.207(54), and 1.222(60) radians for the cases of the 10mm thick PS-I without an absorber, the 10mm/5mm thick PS-I with the absorber (T=0.49), and the 10mm/2mm thick PS-I with the absorber (T=0.21), respectively.

In the theoretical predictions, the obtained phase shift is associated with the solid angle subtended by the closed curve of the cyclic evolution on the sphere, when this phase shift has a true geometric property. The solid angles, Ω, for three combinations of phase shifters-I and absorbers are given by Eq.(5). Figure 4 shows the extent of the quantitative agreement between the measured values (squares) and the expected values of the geometric phase shift (solid line) for $\sigma = 1/2$ in Eq.(6). A slight deviation from the theory may be due to the fact that the dynamical phase factor was not exactly zero under our experimental conditions.

4. Concluding Remarks

We have derived the geometric phase for the split-beam experiment, which is independent of the spin or polarization of the beam. All results obtained thus far are in complete agreement with the theoretical predictions. It is clear that the geometric phase for the split-beam experiment is proportional to the solid angle subtended by the curve at the origin and that its coefficient is one half. This is due to the fact that the cycle of the transformation in the split-beam experiment is a sequence of SU(2) transformations,[16] i.e., rotations, which is closely related with the spin one half system.[17] Since the geometric phase in the split-beam experiment is independent of the spin of the beam, the same results could be obtained with any kind of particle beam, such as photons, x-rays, atoms, etc.

In an example of the spinor rotation in a homogeneous magnetic field,[5] the Hamiltonian in the rest frame provides a positive energy for a spin-down state, which induces a negative phase shift. This phase shift, however, can be regarded as positive due to the 2π-periodicity of the phase. In other words, while the positive energy due to the Hamiltonian causes a negative phase shift in naive considerations, it can be considered to cause a positive phase shift as well. Thus, a positive and a negative energies, as well as phase shifts, being intuitively regarded to cancel each other, can bring forth an additional phase factor, namely, the geometric phase factor, in the recombined beam. The 2π-periodicity of the phase reflects the geometric phase factor.

This is the first application of the four-plate neutron interferometer for a fundamental measurement. This type of interferometer can be used both for photons and neutrons and is well suited for other fundamental physics applications.

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