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A Neutron Interferometric Test for Quaternion Quantum Mechanics

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We present preliminary results of an on-going neutron interferometric search for evidence for a quantum mechanics based upon the quaternion number field. Following a proposal of Klein, the experiments look for non-commutative quaternionic modifications in the interaction of a neutron with pairs of the known fundamental forces (strong, electromagnetic and gravitation) taken in permuted order.

KEYWORDS: quaternion, quantum mechanics, neutron interferometry

1. Introduction

In 1936, Birkhoff and von Neumann[1] were unable to find any natural hypotheses that quantum theories must be described by complex quantum mechanics. Since then, it has been shown that a mathematically self-consistent quantum theory is possible in a Hilbert space over the quaternionic number field, **Q**, and presents a viable alternative to the standard complex formulation.[2,3]

Quaternions[4] are hyper-complex numbers of the form $Q = q_0 + q_1\hat{i} + q_2\hat{j} + q_3\hat{k} = q_0 + \boldsymbol{J}$ with $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = -1$ and $\hat{i}\hat{j} = -\hat{j}\hat{i} = \hat{k}$ etc., which satisfy associative, distributive but not commutative laws. They form a number field which is closed under addition and multiplication and are the only generalisation of complex numbers for which division is possible and unique. In general quaternions do not commute, however, quaternions having the same $\boldsymbol{J}$ can be written as $Q = a + b\boldsymbol{J}$ and do commute, behaving just like complex numbers. Such quaternions are said to be collinear.

A purely imaginary quaternion taking the place of the complex i in the Schrödinger equation implies the existence of completely new physical phenomena. At the very least, the extra dimensionality of the vector (imaginary) part of the quaternion algebra predicts subtle modifications to existing quantum theories. These include, extra polarisation states of fundamental particles, a new curvature of connections and form of Berry's geometric phase accumulated around any closed trajectory in a curved Q-space (with similar corrections to the topological Aharonov-Bohm effects), and new results to Bell type experiments of multi-particle correlations in entangled states involving tensor products.[5]

Whether quaternionic quantum mechanics (QQM) has any physical manifestations in the real world is a matter for experiment to decide. To date such a QQM has remained well hidden from experimental determination, indicating that any effect is extremely small. The purpose of this search is not so much to observe evidence of a quaternionic effect, but to put an upper limit on the size of any quaternionic term modifying the standard quantum mechanics.

2. Experiment

This work turns to particle interferometry and a re-examination of the interaction of non-relativistic matter-waves with the predicted quaternionic potentials of the classical fields.[6-8] Not knowing the exact form that a quaternionic potential would take, the rationale for experiment is based on the observation that quaternion algebra is non-commutative with the inference that quantum mechanics will behave in a similar way. The basis of such an experiment is that, in a complex theory, the imaginary part of a plane wave transmitted through a pair of square potential barriers is independent of the order of the barriers. However, in QQM, if the operators are not collinear the operators rotate the imaginary quaternionic components of the wave about quaternionic directions normal to the Argand plane (the plane determined by the operators), and give rise to non-commutative phase shifts, as will now be explained.

Consider the theoretical interferometer shown in Fig. 1, a pair of identical quaternionic phase shifters (potentials) are placed in reversed order in each arm. This situation gives rise to an intensity of

$$I = \frac{I_0}{4}\left| e^{\nu \boldsymbol{n}} e^{\mu \boldsymbol{m}} + e^{\mu \boldsymbol{m}} e^{\nu \boldsymbol{n}} \right|^2$$
$$= \frac{I_0}{4}\left| \begin{matrix} (\cos\nu + \boldsymbol{n}\sin\nu)(\cos\mu + \boldsymbol{m}\sin\mu) \\ + (\cos\mu + \boldsymbol{m}\sin\mu)(\cos\nu + \boldsymbol{n}\sin\nu) \end{matrix} \right|^2, \tag{1}$$

where μ and ν are positive numbers and $\boldsymbol{m}$ and $\boldsymbol{n}$ denote unit quaternions of unspecified direction.

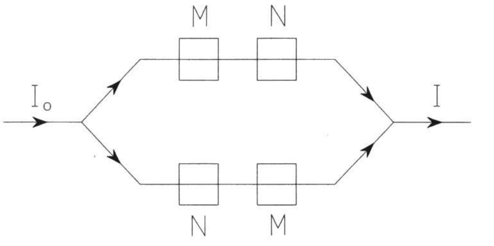

Figure 1. Schematic of the theoretical interferometer used to derive the expected change in intensity due to any non-commutative effect of the ordering of identical quaternionic potentials, M and N, in each arm of the interferometer.

This expressions simplifies to

$$I = I_0[1 - \sin^2 \mu \sin^2 \nu(\boldsymbol{m} \times \boldsymbol{n})^2] \quad . \tag{2}$$

Thus if m and n are non-collinear, there will be a change in intensity due to the reversal of the order of transmission through the two potentials, i.e., a non-commutative effect.

To our knowledge there has only been one previous experiment,[9)] performed by the neutron interferometry group at MURR, following the explicit proposal of Peres,[6)] to look for non-commutative effects in the neutron-nucleus scattering amplitudes. In this experiment neutrons traversed Al and Ti phase shifting slabs inserted in one arm of a neutron interferometer first in one order and then in reverse order. The motivation for selecting Al and Ti is that the real parts of their nuclear scattering lengths are of opposite sign. To better than 1 part in 30000 these two phase shifts commuted. If QQM actually is the theory required to describe the real world, a possible reason for this null result is that the strong interaction operator results in rotation in the same abstract plane, i.e., the quaternion terms are collinear, independent of the scattering nuclei. Note that in this experiment only one fundamental interaction, the neutron-nuclear potential, a manifestation of the strong interaction, was considered.

The present experiment is a generalisation of this previous experiment wherein, following a proposal by Klein,[10)] we subject the split neutron beam of the interferometer to two different fundamental interactions in permuted order. The rationale is that the different fundamental interactions of nature (strong, electroweak and gravitational) may give rise to non-collinear quaternionic operators operating on different quaternionic components of the wavefunctions, i.e., that there may be small but finite angles between the quaternionic directions that the operators rotate about, giving rise to non-commutative phase shifts.

A schematic of the experimental set-up to achieve this is shown in Fig.2. Nominally 1.2Å wavelength neutrons are incident on a perfect single Si crystal neutron interferometer in which, as standard practice, an interferogram is produced as a function of the difference in path length through a nuclear interaction in each arm of the interferometer. Using a special two-section aluminium phase rotator (whose two sections are of different thickness) that straddles the middle blade of the interferometer, two interleaved interferograms can be made as the phase rotator oscillates between incrementing angles, δ, alternating between positive and negative directions (an example of such an interferogram is shown in Fig.3.). In this geometry, the nuclear interaction in the ABD path (path II) of the interferometer alternates between being in front of and behind the middle blade in the positive (+)δ and the minus (-)δ orientations respectively. The reverse is true along path ACD (path I). The second interaction is then introduced in a fixed position near the center blade of the interferometer (before or after) so that the permutation of the order of the nuclear potential with the second potential is achieved for positive and negative δ.

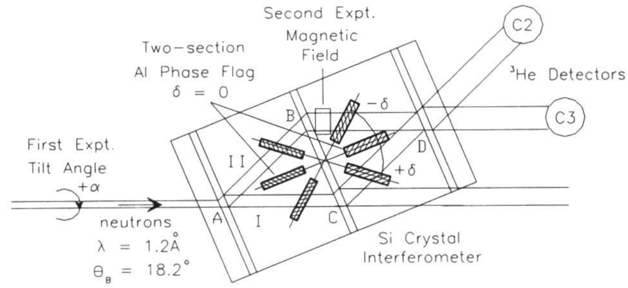

Figure 2. Schematic of the experiment, showing the neutron interferometer (blade separation 35mm, blade thickness 2.5mm) and the two-section phase rotator. The first experiment introduced a gravitational interaction by tilting the interferometer about the incident beam. The second placed a region of magnetic field in path II just after the middle blade.

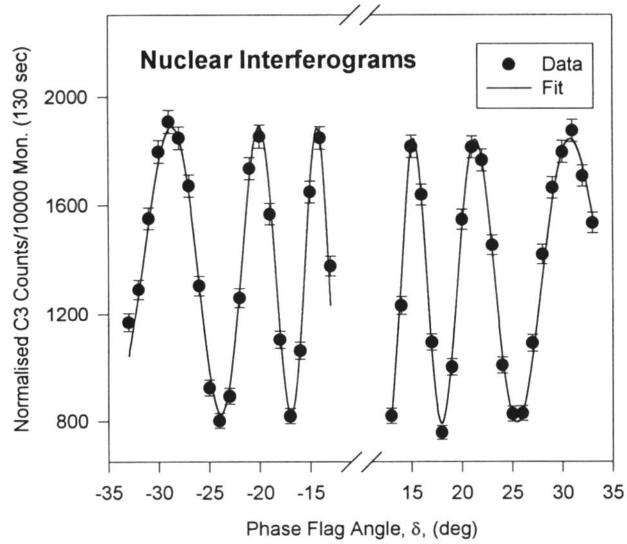

Figure 3. Plot of the interferograms achieved as a function of the angle of the nuclear interaction of the split Al phase flag.

In the first instance, a difference in gravitational potential is introduced by tilting the interferometer through an angle α about the incident beam. An example of this gravitationally induced quantum interference as a function of tilt angle is shown in Fig.4. In the -δ orientation, the split neutron wavepacket in path II traverses a gravitational potential gradient (positive or negative for a positive or negative) along the entire length AB upstream of the nuclear potential of the phase flag in the horizontal section BD. The wavepacket on path I does the reverse, crossing the nuclear potential in the horizontal section AC and then the gravitational potential gradient along CD.

The entire situation is reversed for the +d condition. On path II, the all-permeating gravitational potential gradient still occurs along the length AB, however now the nuclear interaction of the phase flag is part way along this length, resulting in some of the gravitational interaction being downstream of the nuclear potential. Similarly on path I, both interactions are found on CD with some of the gravitational interaction upstream of the nuclear. To this extent the permutation of interactions in each path has been

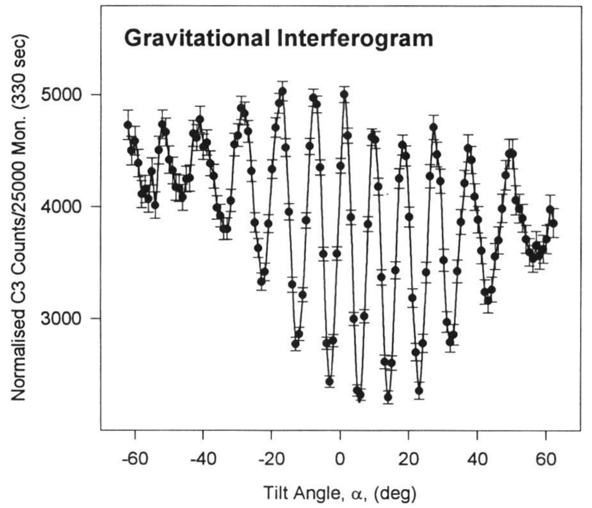

Figure 4. Gravitationally induced quantum interference achieved by tilting the interferometer about the incident neutron beam.

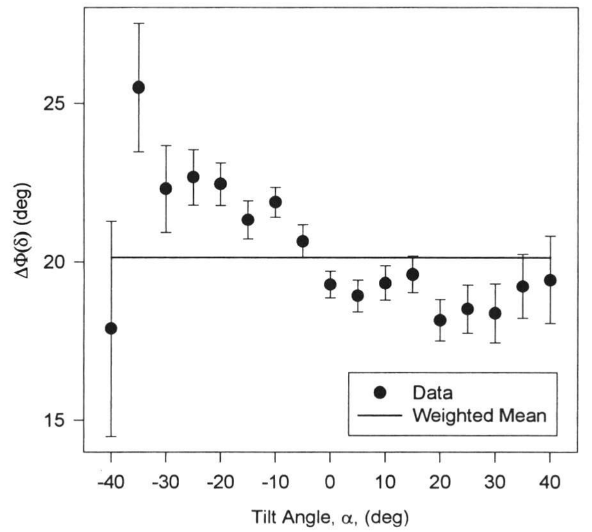

Figure 5. Plot of the difference in offset phases of the $+\delta$ and $-\delta$ interferograms, $\Delta\Phi(\delta)=(\Delta\phi_+ - \Delta\phi_-)/2$, as a function of interferometer tilt angle α.

reversed and a phase shift between the interferograms for each condition is sought.

The $+\delta$ interferogram is given by

$$I_{3+}(\alpha,\delta) = a_{3+} + b_{3+}\cos[\Delta\Phi_{grav}(\alpha) + \Delta\Phi_0 + \Delta\Phi_{nuc}(\delta) + \Delta\Phi_+(\delta)], \quad (3a)$$

and the -d interferogram is given by

$$I_{3-}(\alpha,\delta) = a_{3-} + b_{3-}\cos[\Delta\Phi_{grav}(\alpha) + \Delta\Phi_0 - \Delta\Phi_{nuc}(\delta) + \Delta\Phi_-(\delta)], \quad (3b)$$

where $\Delta\Phi_{grav}(\alpha)$, is the gravitational phase shift, $\Delta\Phi_0$, the original offset phase of the (empty) interferometer, $\Delta\Phi_{nuc}(\delta)$, the nuclear phase shift as a function of aluminium phase rotator angle δ, and $\Delta\Phi(\delta)$, any other phase shift dependent on δ (where a quaternion term would appear). A standard non-linear least-squares fit of the form

$$I_{3\pm}(\pm\delta) = A_{3\pm} + B_{3\pm}\cos[\Delta\Phi_{nuc}(\pm\delta) - \Delta\phi_\pm] \quad (4)$$

as a function of δ to the interferograms (Fig.3) at each tilt angle results in offset phases

$$\Delta\phi_+ = \Delta\Phi_{grav}(\alpha) + \Delta\Phi_0 + \Delta\Phi_+(\delta) \quad (5a)$$

and

$$\Delta\phi_- = \Delta\Phi_{grav}(\alpha) + \Delta\Phi_0 + \Delta\Phi_-(\delta)\,. \quad (5b)$$

We subtract these to obtain $\Delta\phi_+ - \Delta\phi_- = \Delta\Phi_+(\delta) - \Delta\Phi_-(\delta) = 2\Delta\Phi(\delta)$, where by interleaving the $+\delta$ and $-\delta$ interferograms, we have eliminated any even-symmetry part of $\Delta\Phi(\delta)$, and any phase drift in the empty interferometer phase $\Delta\Phi_0$. A plot of $\Delta\Phi(\delta)$ as a function of a is shown in Fig.5. The results indicate a small, but definite systematic effect dependent on the tilt of the interferometer. The source of this effect is being looked into. Initially, we thought it was possibly an artifact of bending of the phase flag and the support under their own weight as they are tilted. Further improvements to the experiment using a more substantial support have shown this not to be the case.

A second similar experiment has been performed by placing a variable vertical magnetic field, $\boldsymbol{B}$, in path II (after the middle interferometer blade, see Fig.2) and permuting the nuclear potential of the phase flag with it, in the same manner as described previously. The magnetic field adds an extra dimension to the experiment as it interacts with the neutron's magnetic moment, causing its intrinsic spin to precess, while the nuclear and gravitational interactions are spin-independent. The form of an interferogram assuming a polarised incident beam is

$$I_3 = a_3 + b_3\cos[\Delta\Phi_{nuc} + \Delta\Phi_0 + \sigma\Delta\Phi_{mag}]\,, \quad (6)$$

where $\Delta\Phi_{nuc}$ and $\Delta\Phi_0$ are the nuclear and offset phases as defined previously and $\Delta\Phi_{mag}$ is the spin-dependent magnetic phase shift, with $\sigma = \pm 1$ for spin up or down, where up and down are defined as parallel and anti-parallel to the applied magnetic field. For an unpolarised incident beam such as used in this experiment, the total intensity is the sum of the intensities for spin up and spin down, that is

$$\begin{aligned} I_3 &= I_{3\uparrow} + I_{3\downarrow} \\ &= a_3 + b_3\cos[\Delta\Phi_{nuc} + \Delta\Phi_0]\cos[\Delta\Phi_{mag}]\,, \end{aligned} \quad (7)$$

allowing separate control of the spin-independent ($\Delta\Phi_{nuc}$) and spin-dependent ($\Delta\Phi_{mag}$) phases. A plot of the measured amplitude of the nuclear interferograms as a function of magnetic field is shown in Fig.6. The noticeable loss in contrast of these interferograms is due to inhomogenieties in the magnetic field across the beam, and means the amplitude b_3 of the nuclear interferogram also has a field dependence.

The interferogram for $+\delta$ becomes

$$\begin{aligned} &I_{3+}(B,\delta) \\ &= a_{3+} + b_{3+}(B)\cos[\Delta\Phi_{nuc}(\delta) + \Delta\Phi_0 + \Delta\Phi_+(\delta)] \\ &\qquad \times \cos[\Delta\Phi_{mag}(B)] \end{aligned} \quad (8a)$$

,and for $-\delta$,

$$I_{3-}(B,\delta) = a_{3-} + b_{3-}(B)\cos[\Delta\Phi_{\rm nuc}(\delta) + \Delta\Phi_0 + \Delta\Phi_-(\delta)] \times \cos[\Delta\Phi_{\rm mag}(B)] \quad (8b)$$

,where again $\Delta\Phi_\pm(\delta)$ is any other spin-independent phase shift dependent on δ. Fitting aluminium phase rotator scans to this as a function of δ at each magnetic field setting (which only effects the amplitude of the interferogram) results in offset phases

$$\Delta\phi_+ = \Delta\Phi_0 + \Delta\Phi_+(\delta) \quad (9a)$$

and

$$\Delta\phi_- = \Delta\Phi_0 + \Delta\Phi_-(\delta) \quad . \quad (9b)$$

As before, subtracting $\Delta\phi_+ - \Delta\phi_- = \Delta\Phi_+(\delta) - \Delta\Phi_-(\delta) = 2\Delta\Phi(\delta)$. A plot of this difference phase as a function of magnetic field is shown in Fig.7. The results again indicate a small but definite systematic effect dependent on the sign of $\cos[\Delta\Phi_{\rm mag}(B)]$ and size of the magnetic field. No explanation for this effect has as yet been found, but experiments using more uniform and larger field distributions across the beam are planned.

It is also possible to perform magnetic interferograms for fixed values of nuclear phase, wherein the interferogram has the form

$$I_3(B,\delta) = a_3 + b_3(B)\cos[\Delta\Phi_{\rm nuc}(\delta) + \Delta\Phi_0] \times \cos[\Delta\Phi_{\rm mag}(B) + \Delta\Phi(B,\delta)] \quad , \quad (10)$$

where $\Delta\Phi(B,\delta)$ is a spin-dependent phase shift dependent on δ. This is currently being pursued.

3. Conclusion

Preliminary results of this on-going experiment are inconclusive. Any non-commutative effect of quaternionic nature should not result in phase deviations as large as we have found here, otherwise it would have been noted before.[11] It should be noted that the theoretical limit to the observability of truly non-commuting phase shifts is the size of the overlapping nuclear-gravitational interaction, this is at least an order of magnitude below present experimental sensitivity. On-going work to isolate all possible systematic effects is being carried out, and a rigorous conclusion will be reported in a subsequent paper.

Acknowledgments

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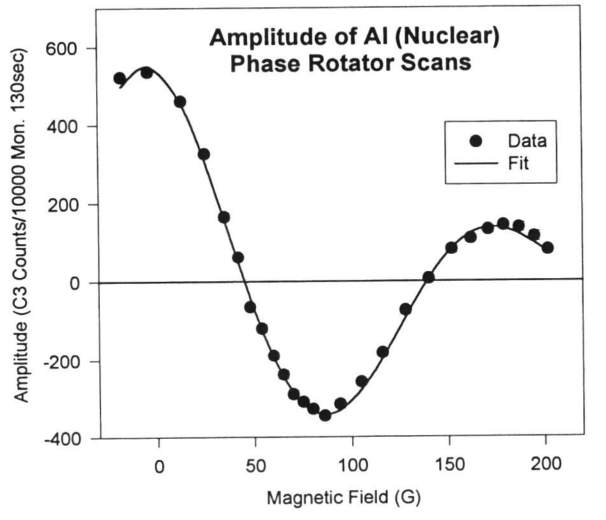

Figure 6. Plot of the amplitude of the nuclear interferograms as a function of magnetic field in one arm of the interferometer. The dramatic loss in contrast is assumed to be due to inhomogenieties in the magnetic field across the beam. The line represents a sinusoidal fit of linearly decreasing amplitude.

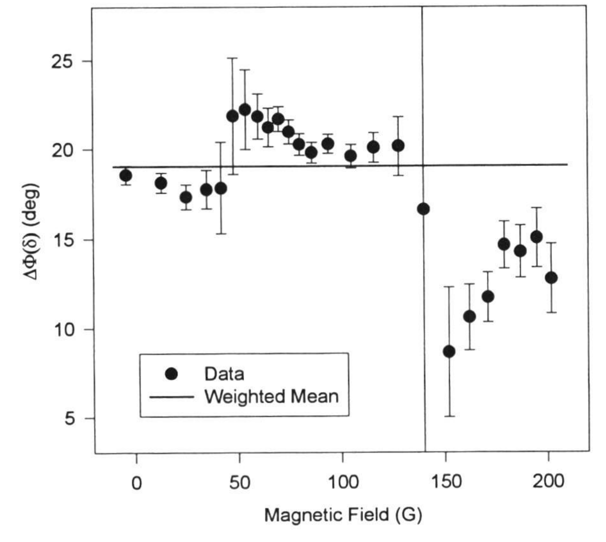

Figure 7. Plot of the offset phase as a function of magnetic field.

1) G. Birkhoff and J. von Neumann, *Ann. Math.* 37, 823 (1936).
2) T. Kaneno, *Prog. Theor. Phys.* 23, 17 (1960).
3) D. Finkelstein, J.M. Jauch, S. Schiminovich and D. Speiser, *J. Math. Phys.* 3 , 207 (1962).
4) W.R. Hamilton, *Elements of Quaternions* (Longmans, Green, & Co., London, 1866).
5) S.P. Brumby, G.C. Joshi and R. Anderson, *Phys. Rev.* A51, 976 (1995).
6) A. Peres, *Phys. Rev. Lett.* 42, 683 (1979).
7) S.L. Adler, *Phys. Rev.* D37, 3654 (1988). For a more detailed description of the entire topic see "*Quaternion Quantum Mechanics and Quantum Fields*" by the same author (Oxford University, 1994).
8) A.J. Davies and B.H.J. McKellar, *Phys. Rev.* A46, 3671 (1992).
9) H. Kaiser, E.A. George and S.A. Werner, *Phys.Rev.* A 29, 2276 (1984).
10) A.G. Klein, *Physica* B151, 44 (1988).
11) C.G. Shull, D.K. Atwood, J. Arthur and M.A. Horne, *Phys. Rev. Lett.* 44, 765 (1980).