Ultracold Neutron Production and Storage in Superfluid ⁴He

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A cross-section model for neutron scattering in liquid ⁴He at temperatures down to 0.1 K is developed, in which some fundamental excitations in superfluid and normal ⁴He are described in terms of phonon-roton (quasiparticle) excitation at temperatures below 2.17 K, density(sound) one at all temperatures up to 4.2 K and recoil scattering from a free ⁴He atom for high incident energies of a neutron. The cross-section model is analyzed quantitatively by comparison with experimental results, both double-differential and total, for many different values of temperature and neutron energy. Scattering cross-sections in the vicinity of 1 meV for temperatures below 1 K are calculated to estimate ultracold neutron production, since a 1 meV neutron can exchange its energy and momentum entirely with those of a created phonon in liquid ⁴He. Furthermore it is shown that lowering temperature below about 0.5 K significantly suppresses upscattering of an ultracold neutron by neutron β -decay only. Consequently the number density of ultracold neutrons attainable at 0.1 K is determined in the case of no significant upscattering and no wall loss. For instance, it is estimated to be 4.2×10^4 cm⁻³ with neutron energies below 0.335 μ eV for an incident neutron flux of 1.9×10^9 cm⁻²s⁻¹ meV⁻¹ at 1 meV.

KEYWORDS: ultracold neutron, liquid helium, superfluid, cold neutron, superthermal source

§1. Introduction

Ultracold neutrons(UCNs) are defined to be neutrons with energies of the order of 0.1 μ eV or less, i.e. in neutron velocity below about 5 m/s. Such a neutron can undergo total reflection at material surface: for instance, ⁵⁸Ni has an effective UCN potential of 0.335 μ eV. Hence UCNs can be accumulated and stored in a closed metallic bottle until they disappear by neutron β -decay with a lifetime of 887.6 s.¹) This may provide a new research field such as fundamental physics studies of neutron β decay and electric dipole moment, UCN scattering from condensed matters and relevant new technologies.²)

For this purpose, it is necessary to produce UCNs as many as possible. Various methods of UCN production have been developed and applied by means of the following concepts: for instance, $^{3)}$ deceleration by the gravity, down-scattering in a cryogenic moderator and use of a mechanical neutron turbine. Among these, use of pure superfluid ⁴He at temperatures T lower than 1 K has been expected to be most promising. The idea is that 1 meV neutrons may be down scattered into UCNs by creating phonons in liquid ⁴He, since there are some advantages of a rigorously zero absorption cross section, a well-defined phonon-roton dispersion relation and no significant upscattering of UCNs at lower T. A number of papers on this subject have been published: for instance, the possibility of a strong UCN source⁴) and the application of UCN storage in a vessel for experiments in fundamental physics.⁵⁾

The purpose of the present paper is to study theoretically the UCN production and storage in superfluid ⁴He. A cross-section model for neutron scattering in liquid ⁴He at any T down to 0.1 K⁶) is fully utilized. A discussion is centered on the UCN production by down-scattering of a 1 meV cold neutron and the maximum density of UCNs to be expected at low T.

§2. Cross Section Model

Neutron scattering in liquid ⁴He at all *T* between 0.1 and 4.2 K has been recently described in terms of a crosssection model.⁶) It is based on the following experimental results.^{7,8} At low momentum transfer κ below 0.8 Å⁻¹, a dynamic structure factor $S(\kappa, \omega)$ shows a sharp peak which broadens with increasing *T* but remains welldefined above $T_{\lambda}=2.17$ K. The peak position persists at almost the same energy for *T* below and above T_{λ} . At higher κ up to about 3.6 Å⁻¹, $S(\kappa, \omega)$ at $T < T_{\lambda}$ has a sharp peak plus a broad component. As *T* increases, the magnitude of the peak is reduced rapidly until it vanishes at T_{λ} . The broad one still exists above T_{λ} as being nearly independent of *T*.

The above experimental findings may be interpreted microscopically in terms of a model consisting of two kinds of excitations:⁹⁾ a density-mode excitation related a non-condensate component and a quasiparticle (phonon-roton) excitation due to a condensate component. In superfluid ⁴He, the density and quasiparticle excitations are strongly coupled through the Bose condensate, which leads to a common phonon-roton dispersion curve with an energy $\hbar \omega_R(\kappa)$. At low κ , the sharp excitation arises from a collective density mode among all the atoms lying in and above the condensate. At higher κ , the sharp peak in $S(\kappa, \omega)$ comes from the quasiparticle excitation while the broad one from the density mode. These two excitations are coupled through the Bose condensate fraction $\Delta(T) = 0.1\{1 - (T/T_{\lambda})^{3/2}\}$ for $T \leq T_{\lambda}$

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and = 0 for $T > T_{\lambda}$.⁹⁾

Consequently the double-differential cross section for neutron scattering in liquid 4 He is expressed as

$$\frac{d^2\sigma}{dEd\Omega} = b^2 \frac{k}{k_0} \Big[\Delta(T)S(\kappa)S_{C,S}(\kappa,\omega) + \{1 - \Delta(T)\}S(\kappa)S_{N,S}(\kappa,\omega) \Big]$$
(1)

where b is the scattering length of a bound ⁴He atom, k and k_0 are, respectively, the wave numbers for scattered and incident neutron, $S(\kappa)$ is the static structure factor,¹⁰⁾ and $S_{X,S}(\kappa,\omega)$ (X = C, N) are the dynamic structure factors for a condensate(X = C) and a noncondensate(X = N) component. Each of $S_{X,S}(\kappa,\omega)$ is further expressed by the sum of down- and up-scattering processes to take into account of the above-mentioned excitations. The cross-section model has been found to be in satisfactory agreement with experimental results, both double-differential^{7,11} and total,^{5,12} for many different values of temperature and neutron incident energy.

§3. Ultracold Neutron Production

For a specific down-scattering for an incident neutron energy $E_0 \sim 1 \text{ meV}$ and T < 1 K, Eq.(1) is reduced to

$$\frac{d^2\sigma}{dEd\Omega} = b^2 \frac{k}{k_0} S(\kappa) \Delta(T) \frac{1}{\pi\hbar} \exp\left[q^2 \alpha c - \hbar \omega^+ / 2k_B T\right]$$

$$\times \frac{q^2 \alpha}{(\omega^+)^2 + (q^2 \alpha)^2} \Big|_{\mathcal{C}} + b^2 \frac{k}{k_0} S(\kappa) \{1 - \Delta(T)\} \frac{1}{\pi\hbar}$$

$$\times \exp\left[q^2 \alpha c - \hbar \omega^+ / 2k_B T\right] \frac{q^2 \alpha}{(\omega^+)^2 + (q^2 \alpha)^2} \Big|_{\mathcal{N}} \qquad (2)$$

where $q^2 = \kappa^2 \{1/S(\kappa) - \hbar \omega_R(\kappa)/(\hbar^2 \kappa^2/2m)\}, \ \omega^+ = -\omega + \omega_R(\kappa)$, and $c = \alpha m/k_B T$ with the mass *m* of a ⁴He atom. The general explicit expression for Eq.(1) together with all the definitions of model parameters such as α are given in ref.6.

Equation (2) indicates two excitations by condensate and non-condensate components and both are peaked when $\omega^+ = 0$, i.e. at an energy transfer $\hbar \omega$ equal to $\hbar\omega_B(\kappa)$. This condition is satisfied at the intersection of $\hbar \omega_R(\kappa)^{(8)}$ and a free neutron dispersion curve as $E_0 = \hbar^2 k_0^2/2m$, which is shown in Fig.1. It can be seen from Fig.1 that only a 1 meV neutron can meet with the condition to be able to exchange its energy and momentum entirely with those of a created phonon. Furthermore, lowering T leads to smaller value of phonon width $2q^2\alpha$,⁹⁾ thus producing stronger UCNs. This is significant in the condensate component rather than in the non-condensate one, since the former has much longer lifetime of a phonon (i.e. smaller width $2q^2\alpha$). Consequently it follows that superfluidity at low T plays an essential role in producing UCNs.

The inelastic scattering processes in liquid ⁴He at 1 K are shown in Fig.2 for $E_0 = 1.03$ meV and 0.1 μ eV. The former indicates an UCN production at $\hbar\omega = 1.03$ meV and $\kappa = 0.704$ Å⁻¹, while the latter shows an inverse process as upscattering of an UCN into a cold neutron. As has been recognized,⁹⁾ lowering T reduces the number



Fig. 1. Dispersion curves for liquid ⁴He and a free neutron, together with the regions covered by the laws of energy-momentum for neutron incident energies $E_0 = 1$ meV, 0.3 meV and 0.1 μ eV.

of phonons in liquid ⁴He, thus resulting in no significant UCN upscattering. This is clearly shown in Fig.3 in terms of total scattering cross sections at many different temperatures.^{5,12} Also shown in Fig.3 is an effective absorption cross section due to the neutron β -decay with a lifetime of $\tau_{\beta} = 887.6 \text{ s.}^{1}$ It is obvious that lowering Tbelow 0.8 K results in much less(or negligible) upscattering of UCNs. Namely there is a possibility to obtain a high density of UCNs.



Fig.2. Inelastic scattering in liquid ⁴He at 1 K for $E_0 = 1.03$ meV(top) and 0.1 μ eV(bottom).



Fig.3. Total scattering cross sections $\sigma_s(E_0)$ of liquid ⁴He and absorption cross section due to neutron β -decay.

Figure 4 shows a total cross section of UCN production defined by

$$\sigma_U(E_0) = \int_0^{E_U} dE \int_{4\pi} d\Omega \frac{d^2 \sigma(E_0 \to E, \mu)}{dE d\Omega}$$
(3)

where $E_U = 0.335 \ \mu \text{eV}$ is taken. As T is decreased, the peak magnitude of $\sigma_U(E_0)$ increases, but the peak area given by an integral of $\sigma_U(E_0)$ with respect to E_0 around 1 meV remains almost constant. For instance, it is $1.17 \times 10^{-6} \text{ b} \cdot \text{meV}/\text{atom at } 0.1 \text{ K}$ where $\sigma_s(E_0) =$ $3.48 \times 10^{-2} \text{ b}/\text{atom}$ and $\sigma_U(E_0) = 5.45 \times 10^{-5} \text{ b}/\text{atom}$ at $E_0 = 1.03 \text{ meV}$. It follows from this that an UCN production is an extremely rare event with a mean free path of about 5 km in liquid ⁴He at 0.1 K.



Fig.4. Total scattering cross sections $\sigma_U(E_0)$ of UCN production for liquid ⁴He at 0.1, 0.5, 1.0 and 1.5 K.

§4. Ultracold Neutron Storage

The maximum UCN density to be expected may be evaluated as follows. Assuming that an UCN disappears only by neutron β -decay, not by UCN loss due to upscattering with a phonon, leakage from a liquid ⁴He system and absorption by impurities, then it is possible to determine the energy spectrum $\rho(E)$ of UCNs:

$$\frac{1}{\tau_{\beta}}\rho(E) = \int dE_0\phi(E_0) \int_{4\pi} d\Omega N_0 \frac{d^2\sigma(E_0 \to E, \mu)}{dEd\Omega} \quad (4)$$

where $\phi(E_0)$ is the energy spectrum of incident neutrons and N_0 is the number density of ⁴He with 2.2x10²² cm⁻³ at 0.1 K. Hence the total UCN density ρ_U is given by $\rho_U = \int_0^{E_U} \rho(E) dE$. It is estimated to be 4.2x10⁴ UCNs/cm³ at 0.1 K for $\phi_0 = 1.9x10^9$ cm²s⁻¹meV⁻¹ at 1 meV (total ϕ_0 of 1.0x10¹⁰ cm²s⁻¹).¹³ This magnitude is about 500 times larger than the experimentally achieved one of about 90 UCNs/cm³ at ILL.³

§5. Concluding Remarks

Production and storage of UCNs in superfluid ⁴He have been discussed physically and quantitatively. The discussion is based on the cross-section model which has been recently developed for liquid ⁴He at any temperatures down to 0.1 K. The results thus obtained are clear and easy to understand since a concept of a single down-scattering of a 1 meV neutron into an UCN is used.

In the actual development of a superthermal source based on superfluid ⁴He, such a limited situation may not be expected. For this, the following studies are now in progress: generation of a basic data library as groupconstant sets at many different T, a multigroup neutron transport analysis to estimate a maximum UCN density in the source, enhancement of an UCN density by an appropriate cryogenic moderator/reflector such as liquid D_2 and a possibility of magnetic confinement of more UCNs. These results will be reported in the near future.

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